

***Recent Results in Power System Damping Control
and RLC Network Model Order Reduction***

A talk by Nelson Martins, CEPEL

Department of Electrical & Computer Engineering
University of Maryland, October 6, 2015

N. Martins Talk - Part 1

A Modal Stabilizer for the Independent Damping Control of Aggregate Generator and Intraplant Modes in Multigenerator Power Plants

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Outline of Part 1

1. INTRODUCTION

2. PROOF OF CONCEPT

- Multigenerator Plant with Classical Machines against Infinite Bus (MPIB)
- The Modal 2-channel PSS (PSS-2ch): Basic Concepts and Structure
- Analytical results for MPIB with 2-channel PSSs or with standard PSSs

3. LINEAR SIMULATIONS

- The MPIB Test System
- MPIB Results with No PSS, with PSS-std or with PSS-2ch
- Eigenanalysis, Root Locus, Step Response, Sensitivity Analysis
- Balanced and Imbalanced Operating Conditions
- Symmetric or Asymmetric Impacts

4. NONLINEAR SIMULATIONS

- The MPIB Test System and the Applied 1Ø Faults
- PSS Performances Compared for Different Disturbances

5. CONCLUSIONS

1. INTRODUCTION

Oscillation damping control in multigenerator power plants

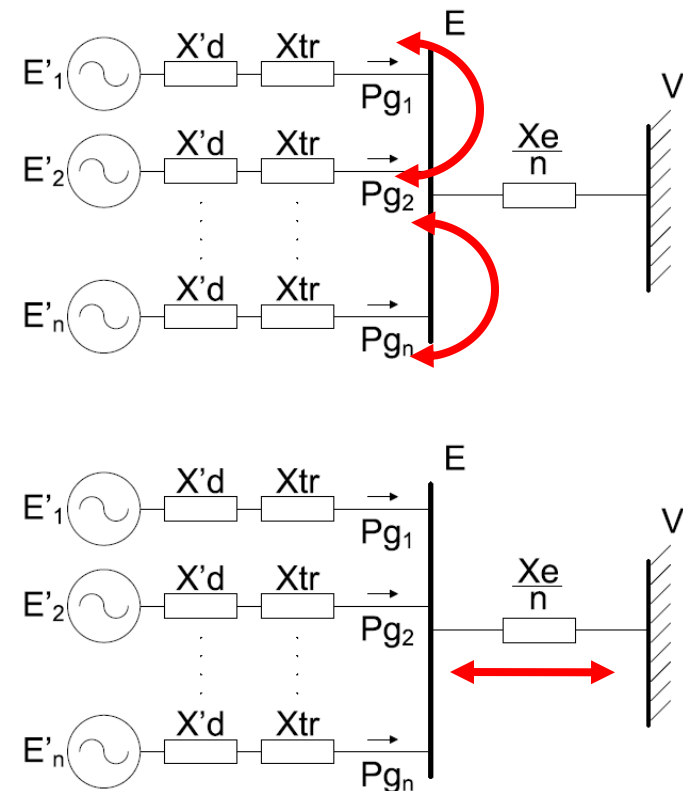
- Types of Electromechanical Oscillations in a symmetric MPIB system:

Intraplant:

- (n-1) identical modes;
- dynamic activity between plant generators
- confined to the plant;

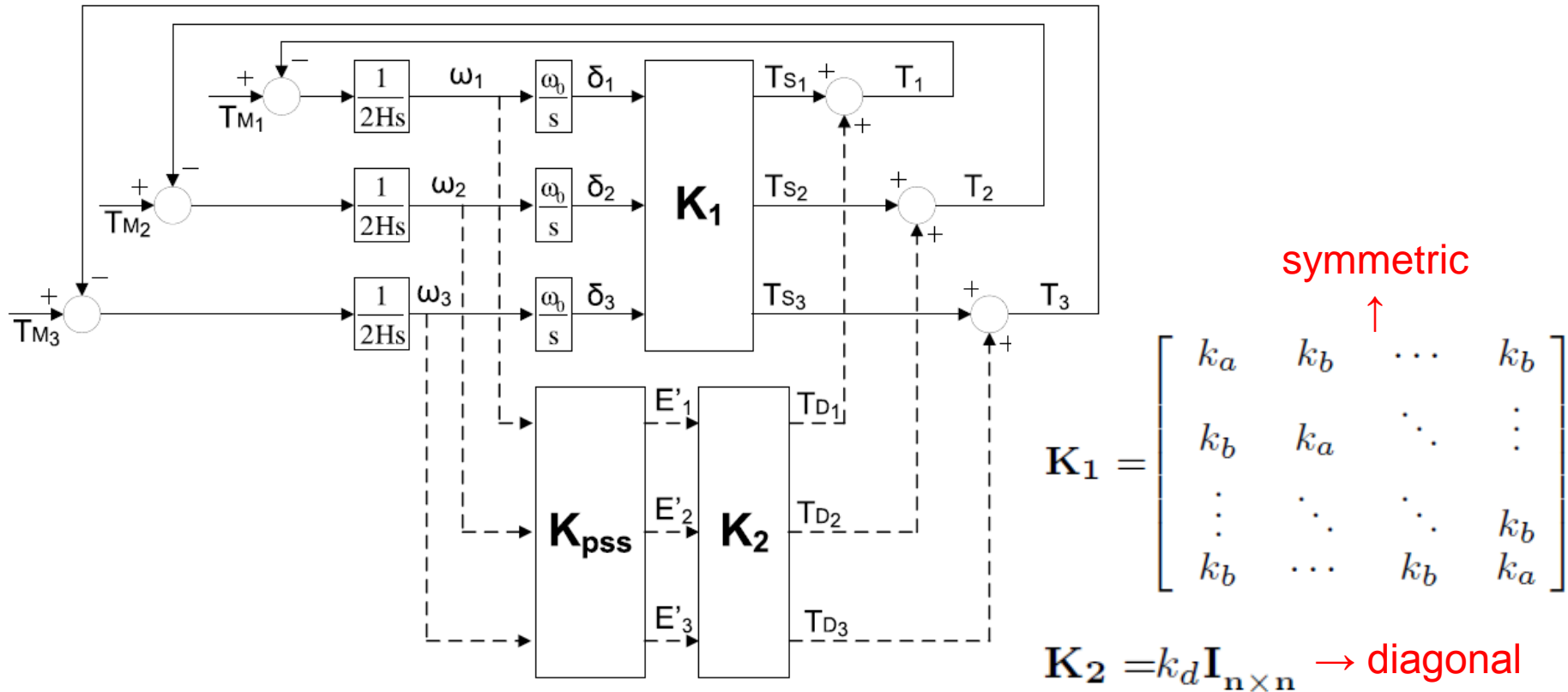
Aggregate:

- 1 mode
- all (n) units oscillate coherently, behaving like a single generator n times larger.
- Related to the all external dynamics (external modes)
- PSS must damp adequately these oscillations



2. PROOF OF CONCEPT

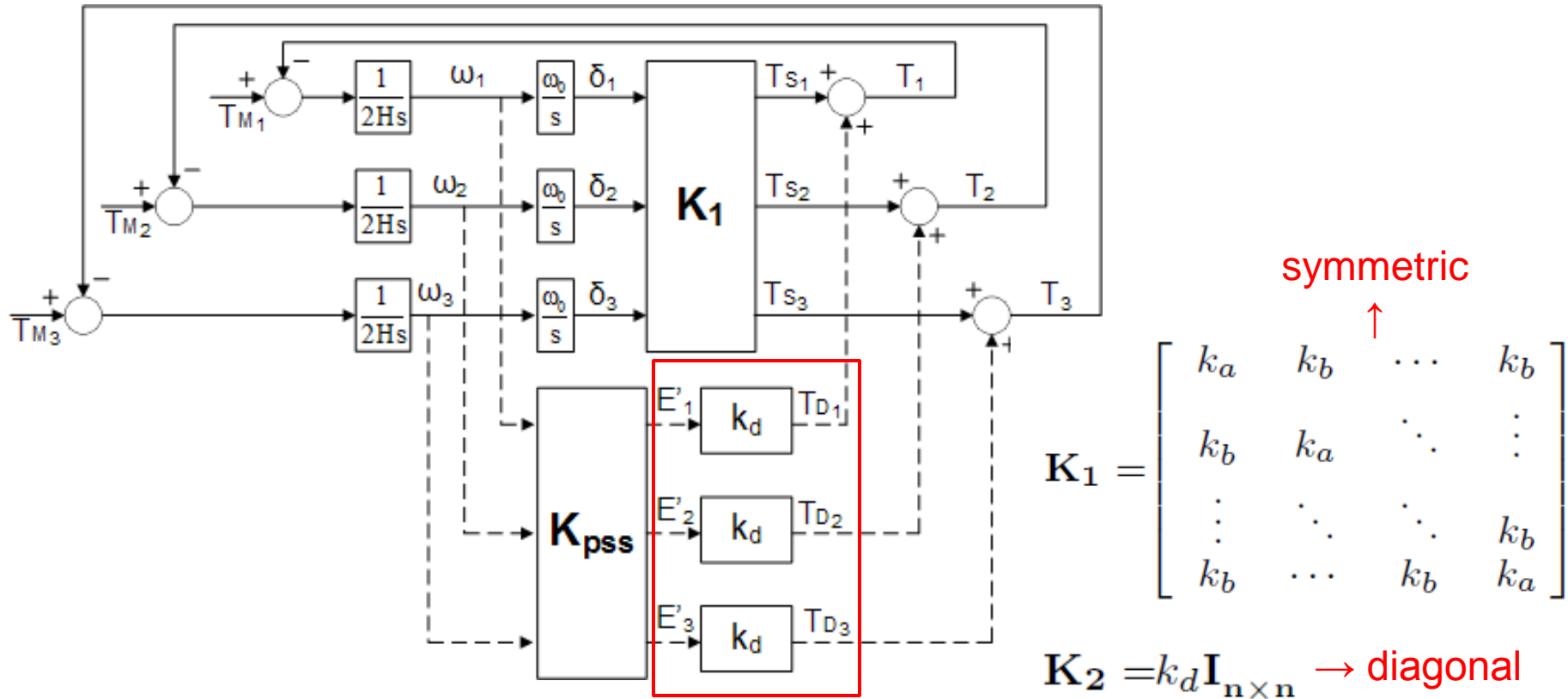
Linear control diagram of MPIB system



- Algebraic analysis described for $n=3$, but results extend to the n -machine case
- Assumptions for simplified analytical study
 - Classical machines (2nd order); all units have equal parameters and loadings (K_1)
 - PSSs are pure gains and induced voltages E' are in phase with own rotor speeds (K_2)

2. PROOF OF CONCEPT

Linear control diagram of MPIB system

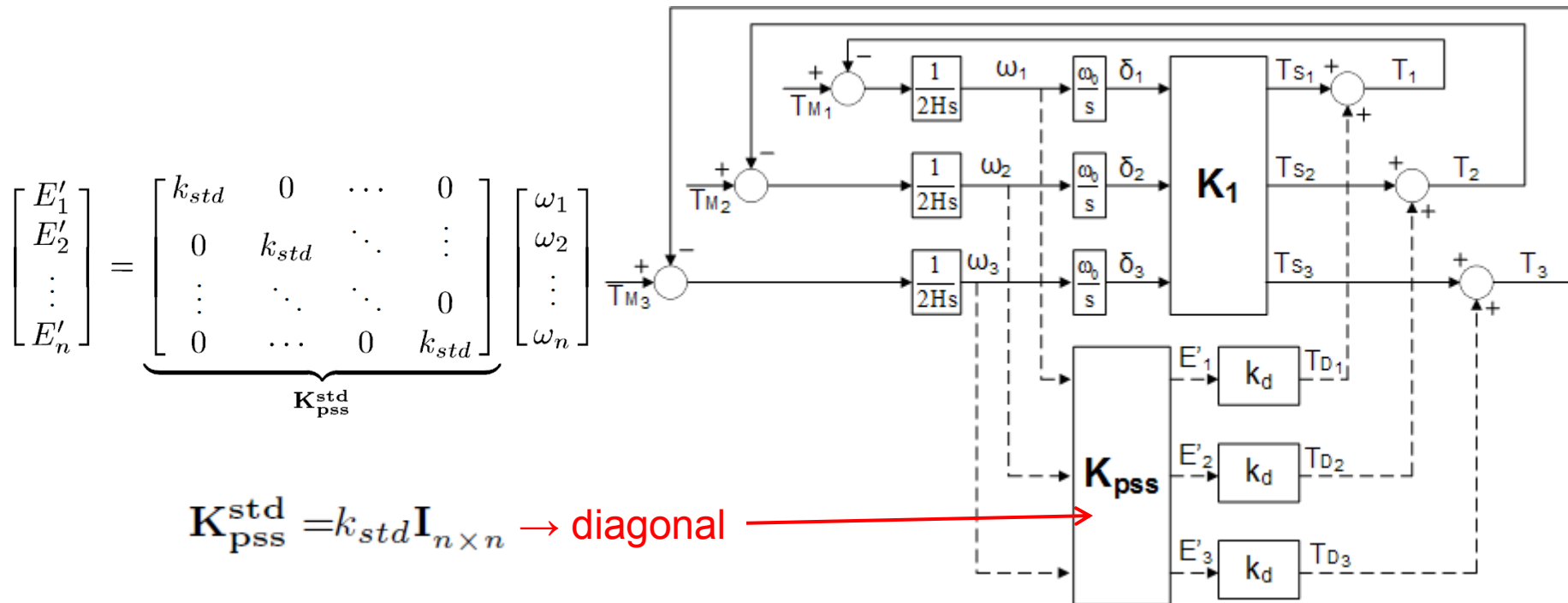


- Algebraic analysis described for **n=3**, but results extend to the **n-machine** case
- Assumptions for simplified analytical study
 - Classical machines (2nd order); all units have equal parameters and loadings (**K1**)
 - PSSs are pure gains and induced voltages E' are in phase with own rotor speeds (**K2**)

2. PROOF OF CONCEPT

MPIB System with Standard PSSs

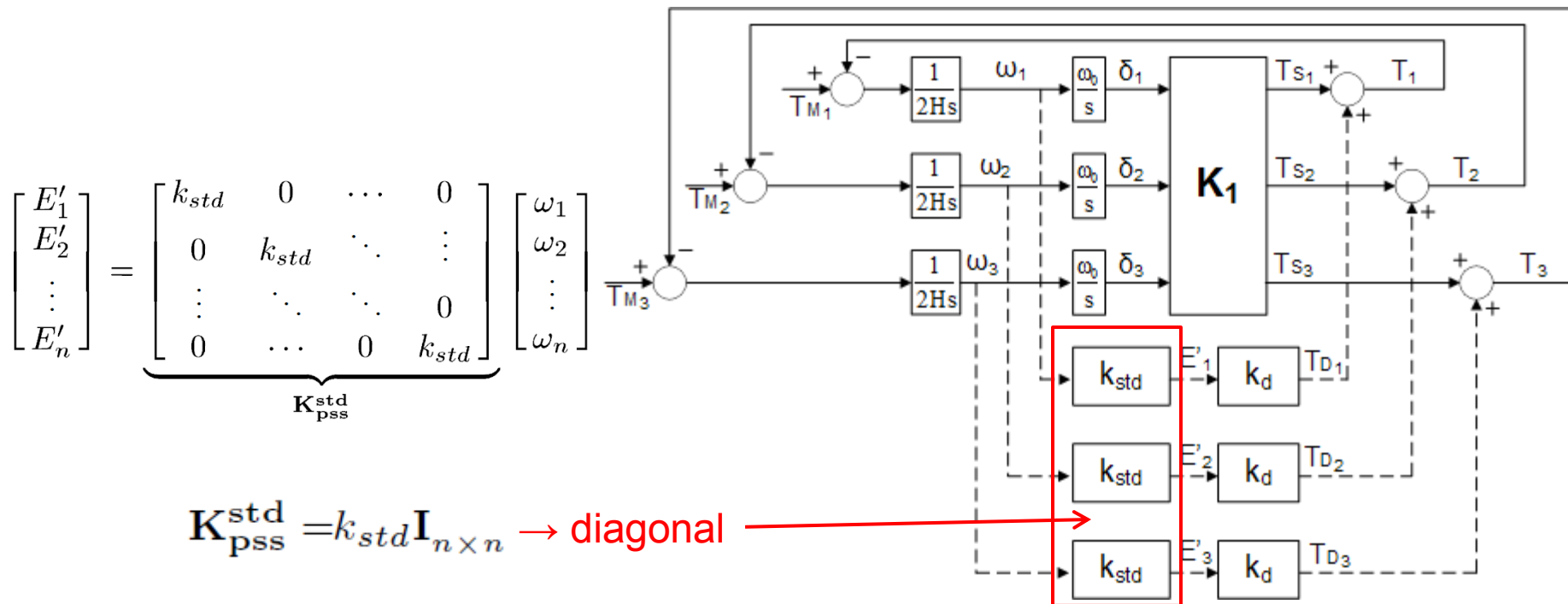
- A standard PSS induces voltage changes that are in phase with its own generator speed (single channel)
- Damps both intraplant and aggregate modes through the same dynamic (phase & gain) compensation channel;
- Their frequencies and damping ratios cannot be set independently.



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2. PROOF OF CONCEPT

MPIB System with Standard PSSs

- State matrix (A^{std}) for the MPIB system equipped with standard PSSs, where the state vector is $X=[\omega_1, \delta_1, \omega_2, \delta_2, \omega_3, \delta_3]$

$$\alpha \triangleq \frac{k_a}{2H} \quad , \quad \beta \triangleq \frac{k_b}{2H} \quad , \quad 2\gamma_{std} \triangleq \frac{k_{std}k_d}{2H}$$

$$A^{std} = \left[\begin{array}{cc|cc|cc} -2\gamma_{std} & -\alpha & 0 & -\beta & 0 & -\beta \\ w_0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & -\beta & -2\gamma_{std} & -\alpha & 0 & -\beta \\ 0 & 0 & w_0 & 0 & 0 & 0 \\ \hline 0 & -\beta & 0 & -\beta & -2\gamma_{std} & -\alpha \\ 0 & 0 & 0 & 0 & w_0 & 0 \end{array} \right]$$

2. PROOF OF CONCEPT

MPIB System with Standard PSSs

- Similarity transformation with matrix \mathbf{P} block-diagonalizes the state matrix \mathbf{A}

$$\bar{\mathbf{A}} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{I}_{m \times m} & \mathbf{I}_{m \times m} & \mathbf{I}_{m \times m} & \cdots & \mathbf{I}_{m \times m} \\ \mathbf{I}_{m \times m} & -\mathbf{I}_{m \times m} & \mathbf{0}_{m \times m} & \cdots & \mathbf{0}_{m \times m} \\ \mathbf{I}_{m \times m} & \mathbf{0}_{m \times m} & -\mathbf{I}_{m \times m} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathbf{0}_{m \times m} \\ \mathbf{I}_{m \times m} & \mathbf{0}_{m \times m} & \cdots & \mathbf{0}_{m \times m} & -\mathbf{I}_{m \times m} \end{bmatrix}$$

$$\bar{\mathbf{A}}^{\text{std}} = \begin{bmatrix} \boxed{\begin{matrix} -2\gamma_{std} & -(\alpha + 2\beta) \\ w_0 & 0 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} -2\gamma_{std} & -(\alpha - \beta) \\ w_0 & 0 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & \boxed{\begin{matrix} -2\gamma_{std} & -(\alpha - \beta) \\ w_0 & 0 \end{matrix}} \end{bmatrix}$$

$$\lambda_{ag} = \lambda_{1,2} = -\gamma_{std} \pm j \sqrt{(\alpha + 2\beta)w_0 - \gamma_{std}^2}$$

$$\lambda_{ip} = \lambda_{3,4} = \lambda_{5,6} = -\gamma_{std} \pm j \sqrt{(\alpha - \beta)w_0 - \gamma_{std}^2}$$

- Changes in gain of standard PSS impact the dampings of **both** *ip* and *ag* modes

2. PROOF OF CONCEPT

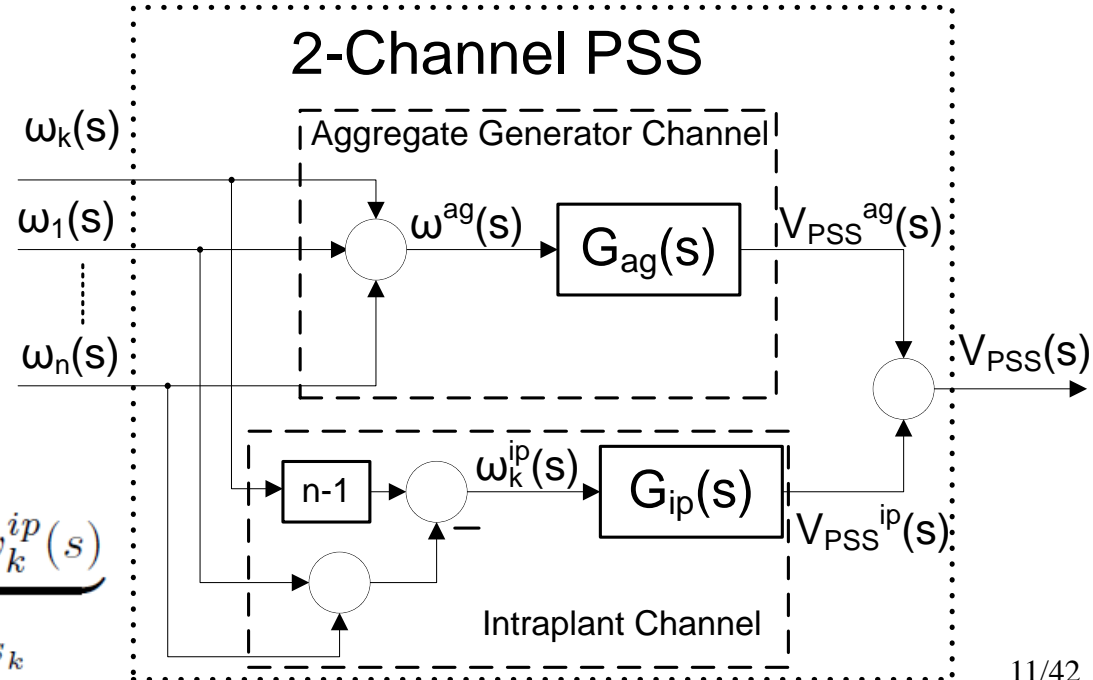
The proposed PSS-2ch

- Damps both oscillation modes with a differential: the intraplant dynamics is kept decoupled from the aggregate dynamics;
- Their frequencies and damping ratios can be independently set
- Output Signal of PSS-2ch has two orthogonal components
 - Aggregate component is equal to the average rotor speed of all (n) units
 - Intraplant: amplified local speed subtracted from speeds of (n-1) parallel units

$$\omega^{ag} = \sum_{i=1}^n \omega_i$$

$$\omega_k^{ip} = (n-1)\omega_k - \sum_{i=1, i \neq k}^n \omega_i$$

$$V_{PSS_k}(s) = \underbrace{G_{ag}(s)\omega^{ag}(s)}_{V_{PSS}^{ag}} + \underbrace{G_{ip}(s)\omega_k^{ip}(s)}_{V_{PSS_k}^{ip}}$$



2. PROOF OF CONCEPT

MPIB System with proposed 2-channel PSSs

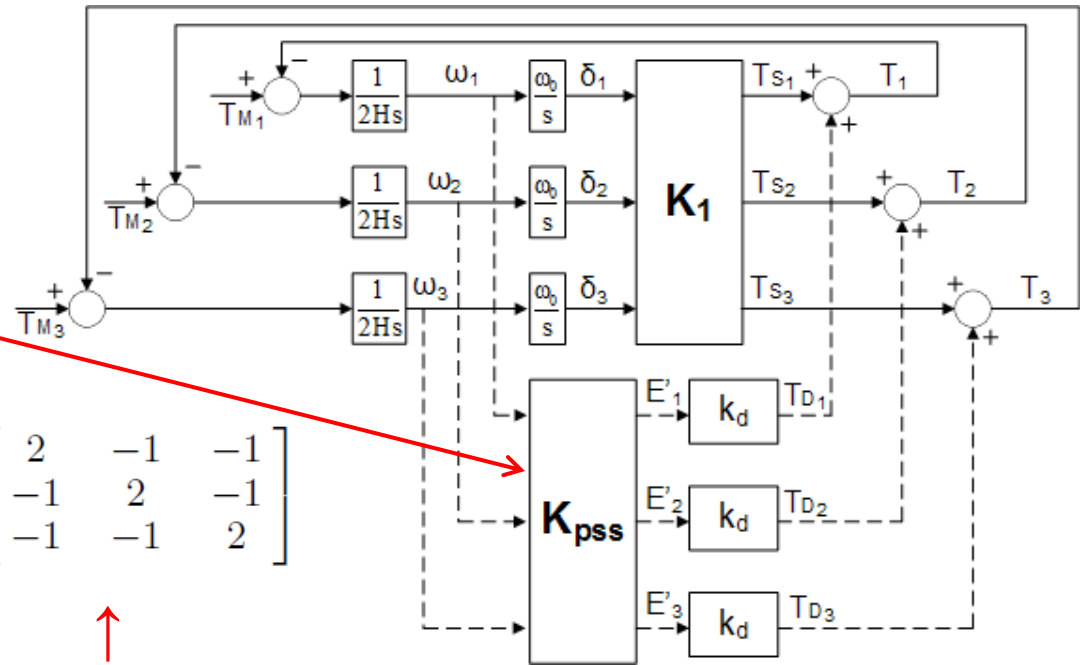
- A 2-channel PSS induces voltage changes that are a smart mix of the speeds from all generator units

$$\begin{bmatrix} E'_1 \\ E'_2 \\ E'_3 \end{bmatrix} = \underbrace{\mathbf{K}_{ag3 \times 3} + \mathbf{K}_{ip3 \times 3}}_{\mathbf{K}_{pss}^{2ch}} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$\mathbf{K}_{ag3 \times 3} = \frac{k_{ag}}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \mathbf{K}_{ip3 \times 3} = \frac{k_{ip}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

↑ n-generator case ↑

$$\mathbf{K}_{ag} = \frac{k_{ag}}{n} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix} \quad \mathbf{K}_{ip} = \frac{k_{ip}}{n} \begin{bmatrix} n-1 & -1 & \cdots & -1 \\ -1 & n-1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & n-1 \end{bmatrix}$$

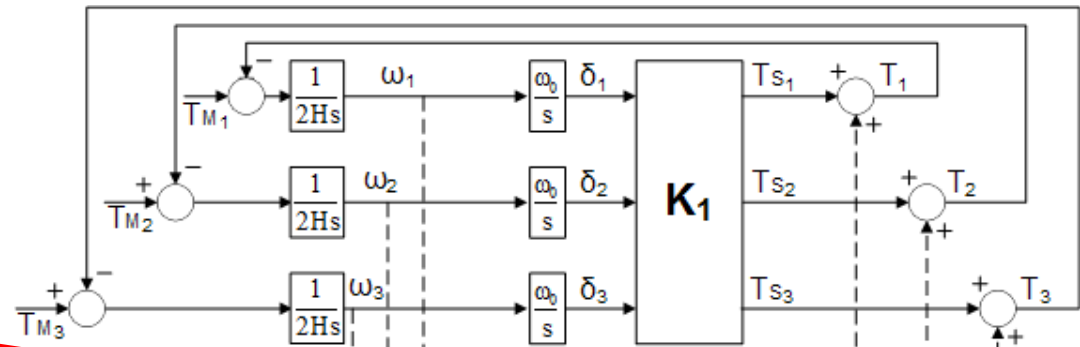


2. PROOF OF CONCEPT

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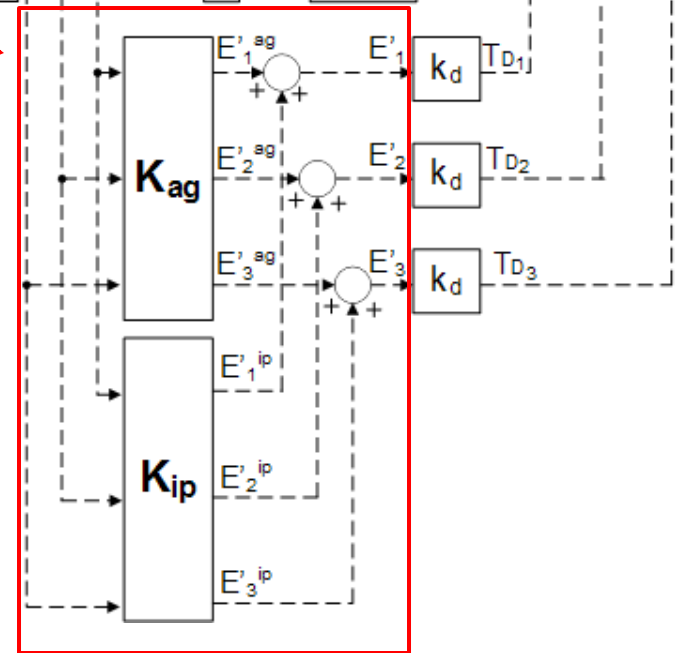
$$\begin{bmatrix} E'_1 \\ E'_2 \\ E'_3 \end{bmatrix} = \underbrace{\mathbf{K}_{ag3 \times 3} + \mathbf{K}_{ip3 \times 3}}_{\mathbf{K}_{pss}^{2ch}} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$



$$\mathbf{K}_{ag3 \times 3} = \frac{k_{ag}}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \mathbf{K}_{ip3 \times 3} = \frac{k_{ip}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

↑ n-generator case ↑

$$\mathbf{K}_{ag} = \frac{k_{ag}}{n} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 1 \end{bmatrix} \quad \mathbf{K}_{ip} = \frac{k_{ip}}{n} \begin{bmatrix} n-1 & -1 & \cdots & -1 \\ -1 & n-1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & n-1 \end{bmatrix}$$



2. PROOF OF CONCEPT

MPIB System with proposed 2-channel PSSs

- State matrix (A^{2ch}) for the MPIB system equipped with 2-channel PSSs, where the state vector is $X=[\omega_1, \delta_1, \omega_2, \delta_2, \omega_3, \delta_3]$

$$\gamma_1 \triangleq \gamma_{ag} + 2\gamma_{ip} \quad , \quad \gamma_2 \triangleq \gamma_{ag} - \gamma_{ip}$$

$$2\gamma_{ag} \triangleq \frac{k_d}{3} \frac{k_{ag}}{2H} \quad , \quad 2\gamma_{ip} \triangleq \frac{k_d}{3} \frac{k_{ip}}{2H}$$

$$A^{2ch} = \left[\begin{array}{cc|cc|cc} -2\gamma_1 & -\alpha & -2\gamma_2 & -\beta & -2\gamma_2 & -\beta \\ w_0 & 0 & 0 & 0 & 0 & 0 \\ \hline -2\gamma_2 & -\beta & -2\gamma_1 & -\alpha & -2\gamma_2 & -\beta \\ 0 & 0 & w_0 & 0 & 0 & 0 \\ \hline -2\gamma_2 & -\beta & -2\gamma_2 & -\beta & -2\gamma_1 & -\alpha \\ 0 & 0 & 0 & 0 & w_0 & 0 \end{array} \right]$$

2. PROOF OF CONCEPT

MPIB System with proposed 2-channel PSSs

- Similarity transformation with matrix P block-diagonalizes the state matrix A

$$\bar{\mathbf{A}}^{2\text{ch}} = \begin{bmatrix} \boxed{\begin{matrix} -2\gamma_{ag} & -(\alpha + 2\beta) \\ w_0 & 0 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} -2\gamma_{ip} & -(\alpha - \beta) \\ w_0 & 0 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & \boxed{\begin{matrix} -2\gamma_{ip} & -(\alpha - \beta) \\ w_0 & 0 \end{matrix}} \end{bmatrix}$$

$$\lambda_{ag} = -\gamma_{ag} \pm j\sqrt{(\alpha + 2\beta)w_0 - \gamma_{ag}^2}$$

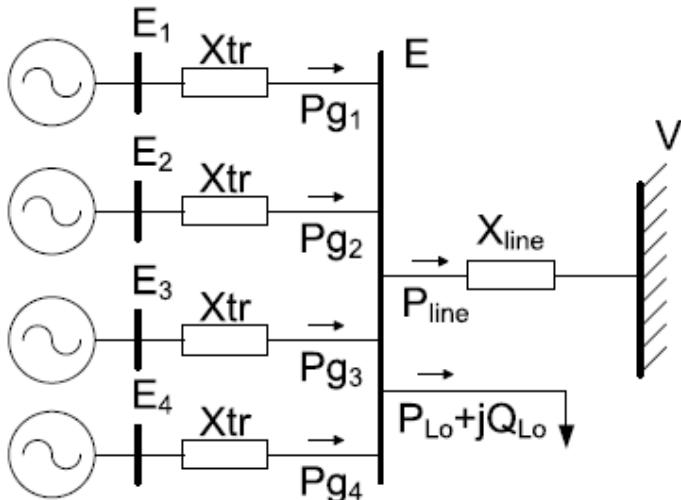
$$\lambda_{ip} = -\gamma_{ip} \pm j\sqrt{(\alpha - \beta)w_0 - \gamma_{ip}^2}$$

- The damping ratios for the intraplant and aggregate modes can be **independently set** by adjusting the gains, either K_{ip} or K_{ag} , of the PSS-2ch.

3. LINEAR SIMULATIONS

MPIB Test System with Slow Response Exciter

- Test system has 4-generator plant and unstable, low frequency “interarea” mode
 - Large const-P load at high-side bus & high impedance transmission line
- Round rotor generator (detailed 6th-order model);
- Slow response excitation system → hinders effective damping role of standard PSSs
- All values are given in pu on the MVA base of a single generating unit



- $S_n=250$ MVA , $H=3.53$ pu
- $X_l=0.16$, $R_a=0.0023$, $X_d=1.81$, $X_q=1.76$
- $X'_d=0.3$, $X'_q=0.61$, $X''_d=0.217$, $X''_q=0.217$
- $T'_{d0}=7.8$, $T'_{q0}=0.9$, $T''_{d0}=0.022$, $T''_{q0}=0.074$
- System base: 250MVA
- Impedances: $X_{tr}=0.1$ pu, $X_{line}=8$ pu
- Voltages: $E_i=1.0$ pu, $E=0.974$ pu, $V=1.0$ pu
- Power Flow: $P_{g_i}=0.96$ pu, $P_{lo}=3.76$ pu (constant P), $Q_{lo}=0.80$ pu (constant Z), $P_{line}=0.08$ pu

$$G_{exc}(s) = \frac{K_A}{(1 + sT_A)}$$

$$K_A = 10, T_A = 0.8$$

3. LINEAR SIMULATIONS

Root Locus for MPIB System with Standard PSSs

$$G^{std}(s) = k_{std} \frac{10s}{(1 + 10s)} \frac{1 + 0.8s}{(1 + 0.2s)} \frac{1 + 0.8s}{(1 + 0.2s)}$$

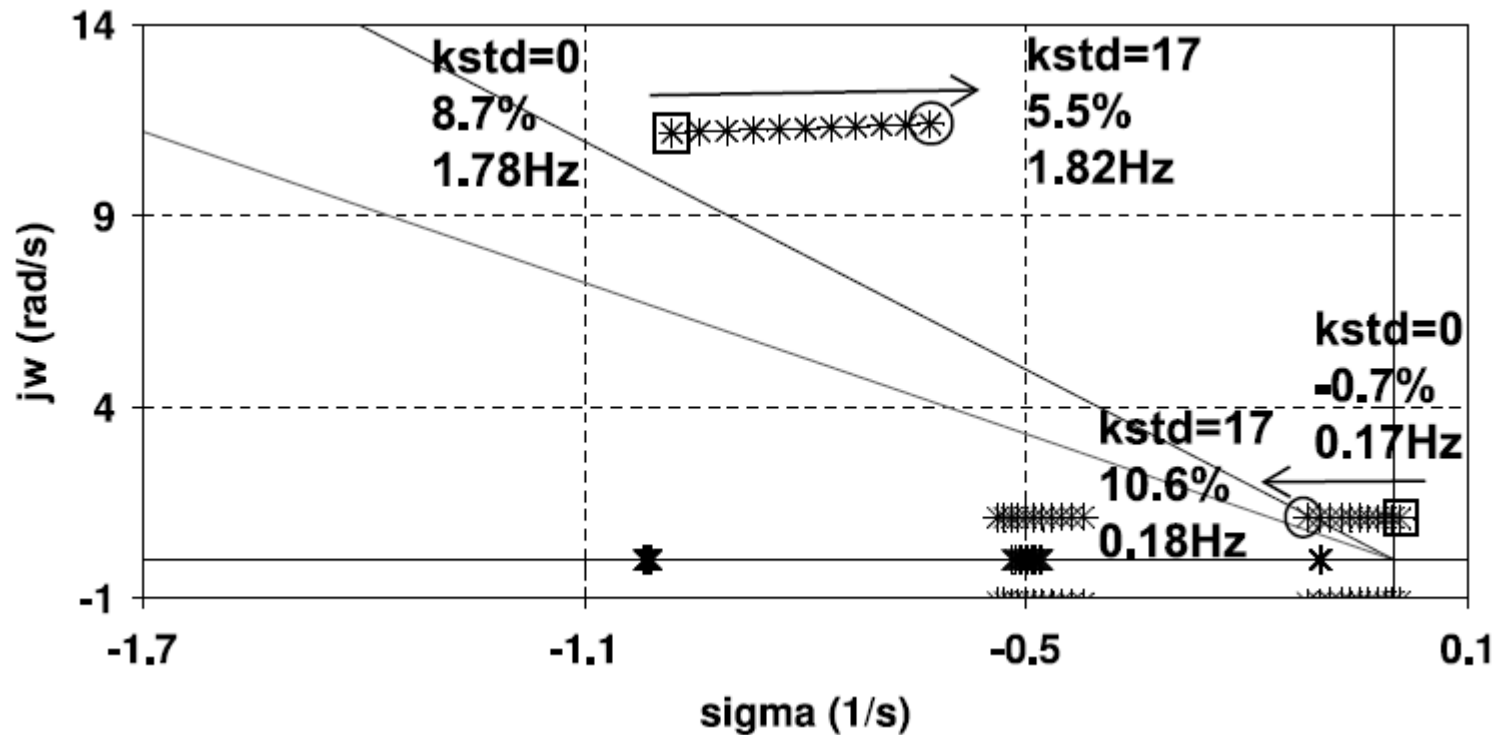


Fig. 21: RL plot for the MPIB Slow-exc system, with the four (standard) PSSs having their gains (k_{std}) varying from 0 up to 17 in steps of 1.7.

3. LINEAR SIMULATIONS

Root Locus for 2-ch PSSs

$$G_{ag}^{2ch}(s) = k_{ag} \frac{10s}{(1+10s)} \frac{1+0.8s}{(1+0.2s)} \frac{1+0.8s}{(1+0.2s)}$$

$$G_{ip}^{2ch}(s) = k_{ip} \frac{2s}{(1+2s)} \frac{1+0.2s}{(1+0.05s)}$$

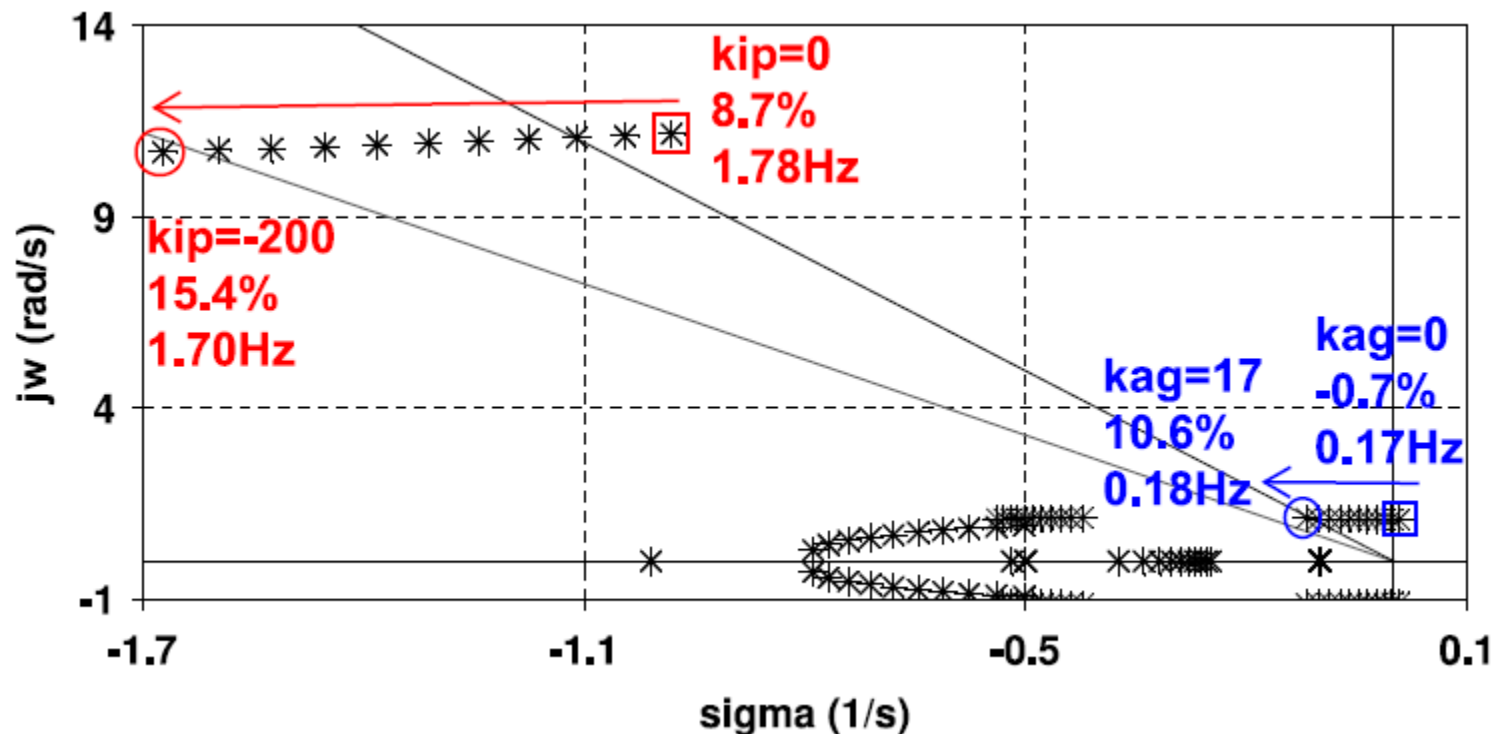


Fig. 20: RL plot of the MPIB Slow-Exc system for the simultaneous variation of the gains of the four 2-channel PSSs. Gain ranges are 0 to 17 for K_{ag} and 0 to -200 for K_{ip} , which vary in steps of 1.7 and -20, respectively.

3. LINEAR SIMULATIONS

Eigenvalue Results for the Standard and 2-channel PSSs

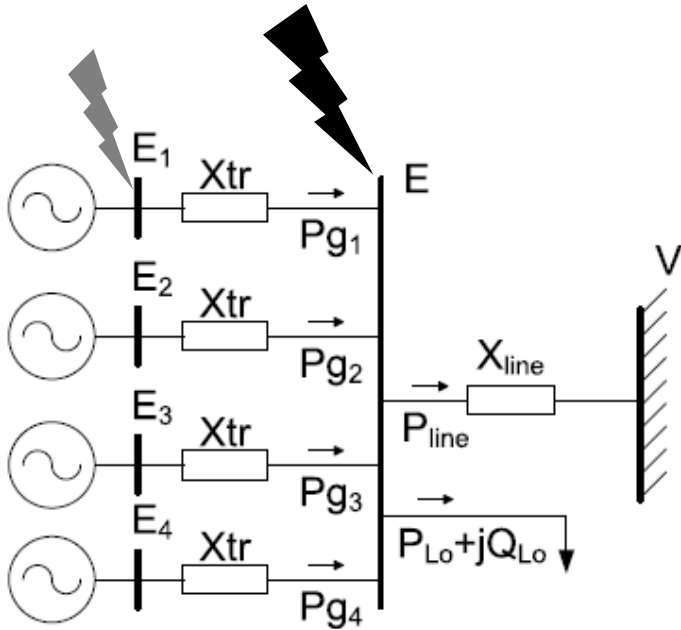
PSS type	Gains
Standard PSS	$k_{std} = 17$
2-channel PSS	$k_{ag} = 17$ & $k_{ip} = -200$

- k_{ag} : confers a damping ratio of 10% to the aggregate mode
- k_{ip} : confers a damping ratio of 15% to the intraplant mode

Modes	Standard PSS	2-Channel PSS	Without PSS
Aggregate Mode	$\omega_d = 0.18Hz$ $\zeta = 10.6\%$	$\omega_d = 0.18Hz$ $\zeta = 10.6\%$	$\omega_d = 0.17Hz$ $\zeta = -0.7\%$
Intraplant Modes	$\omega_d = 1.82Hz$ $\zeta = 5.5\%$	$\omega_d = 1.70Hz$ $\zeta = 15.4\%$	$\omega_d = 1.78Hz$ $\zeta = 8.7\%$

3. LINEAR SIMULATIONS

Types of Disturbance applied to the MPIB System



Symmetric - A disturbance which is applied to bus E , equally impacts all four units, and only excites the aggregate modes.

Asymmetric – A disturbance which is applied to an internal bus (E_1, \dots, E_4) and excites both the aggregate and intraplant modes.

3. LINEAR SIMULATIONS

MPIB System Time Response for Symmetric Disturbance

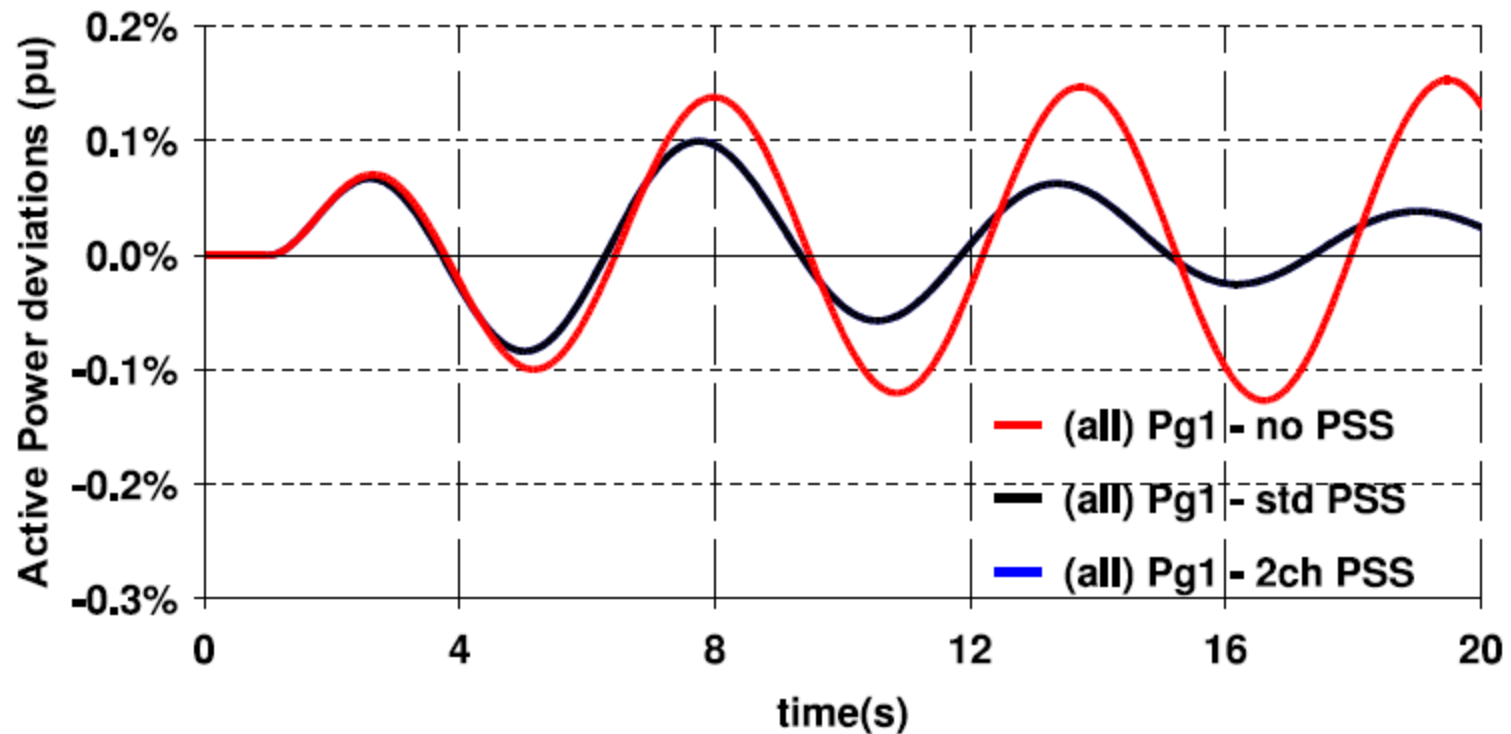


Fig. 22: MPIB Slow-exc system with Small Symmetric disturbance - Active Power responses of one unit.

3. LINEAR SIMULATIONS

MPIB System Time Response for Asymmetric Disturbance

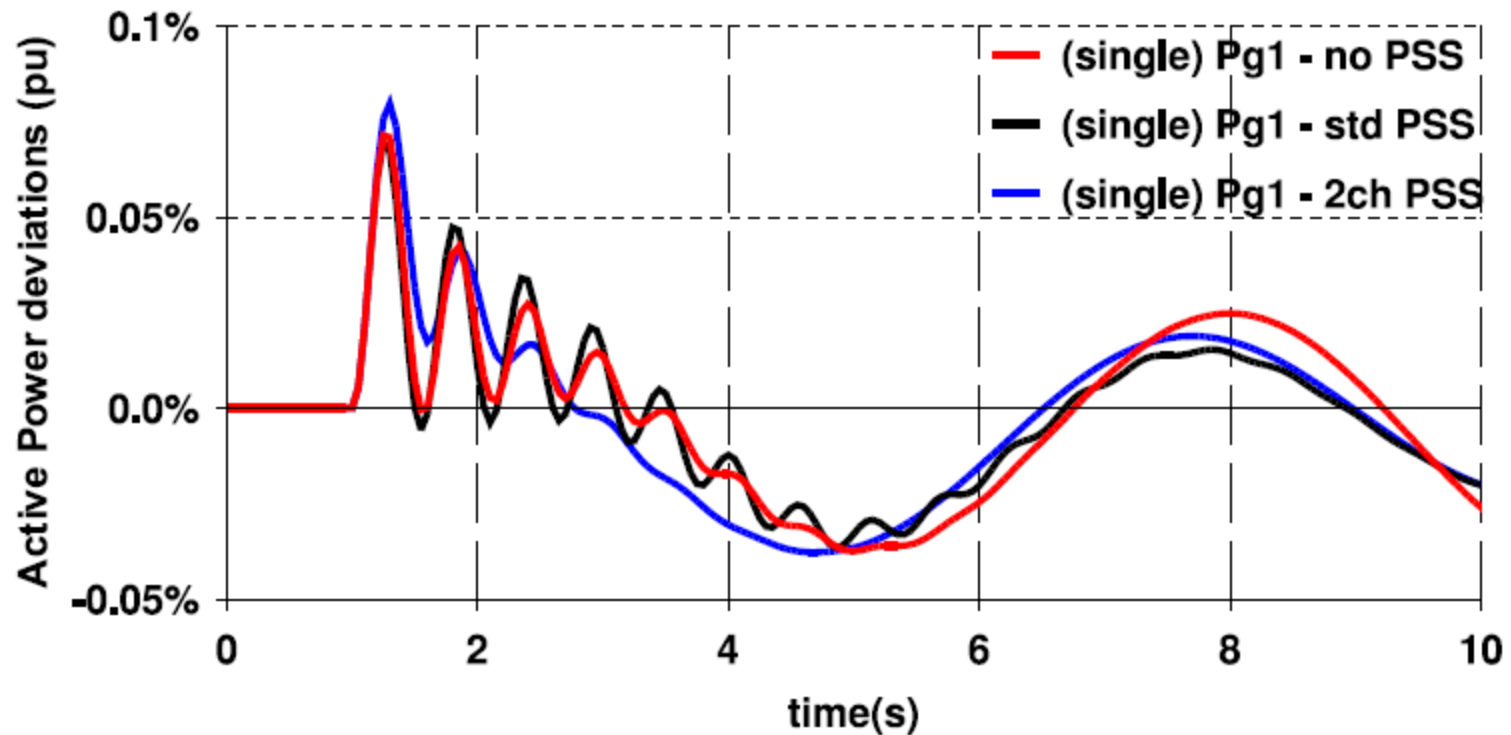


Fig. 23: MPIB Slow-exc system with Small Asymmetric disturbance - Active Power responses of one unit.

3. LINEAR SIMULATIONS

Power Flow and Parameter Data for the Imbalanced MPIB System

- Generator Powers: $P_{g1}=1.00$ pu, $P_{g2}=0.88$ pu, $P_{g3}=0.76$ pu, $P_{g4}=0.64$ pu
- Generator Voltages: $E_1=0.97$ pu, $E_2=0.99$ pu, $E_3=1.01$ pu, $E_4=1.03$ pu
- System: $P_{lo}=3.2$ pu (constant P), $Q_{lo}=0.80$ pu (constant Z), $P_{line}=0.08$ pu
- Slow-exciter gains: $K_{A1}=10$, $K_{A2}=11$, $K_{A3}=9$, $K_{A4}=8$
- Slow-exciter time constants: $T_{A1}=0.8$ s, $T_{A2}=0.7$ s, $T_{A3}=0.9$ s, $T_{A4}=1.0$ s

3. LINEAR SIMULATIONS

Root Locus for Std PSSs in Imbalanced MPIB System

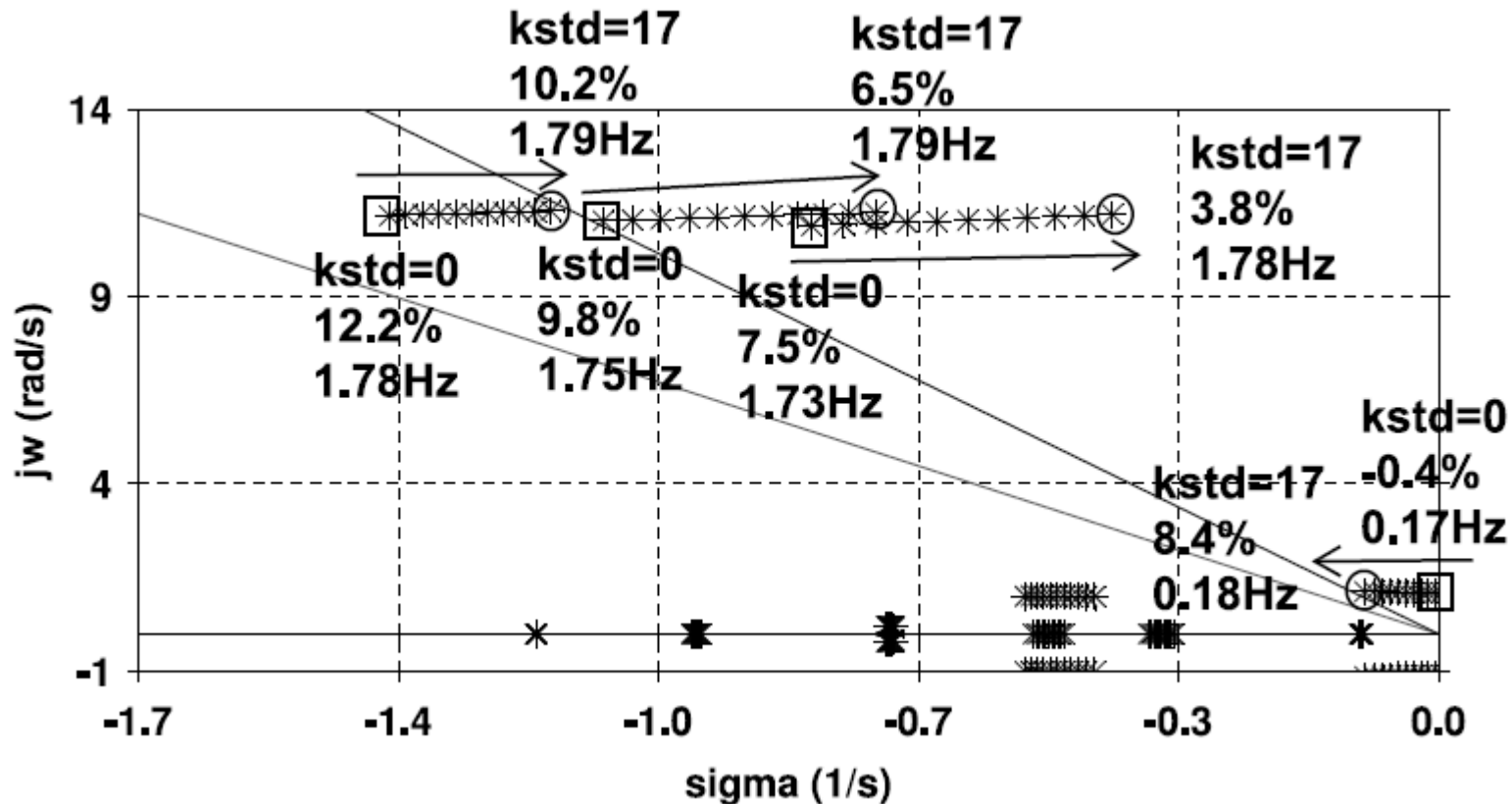


Fig. 24: RL plot for the Unbalanced MPIB Slow-exc system, with the four (standard) PSSs having their gains (k_{std}) varying from 0 up to 17 in steps of 1.7.

3. LINEAR SIMULATIONS

Root Locus for 2-ch PSSs in Imbalanced MPIB System

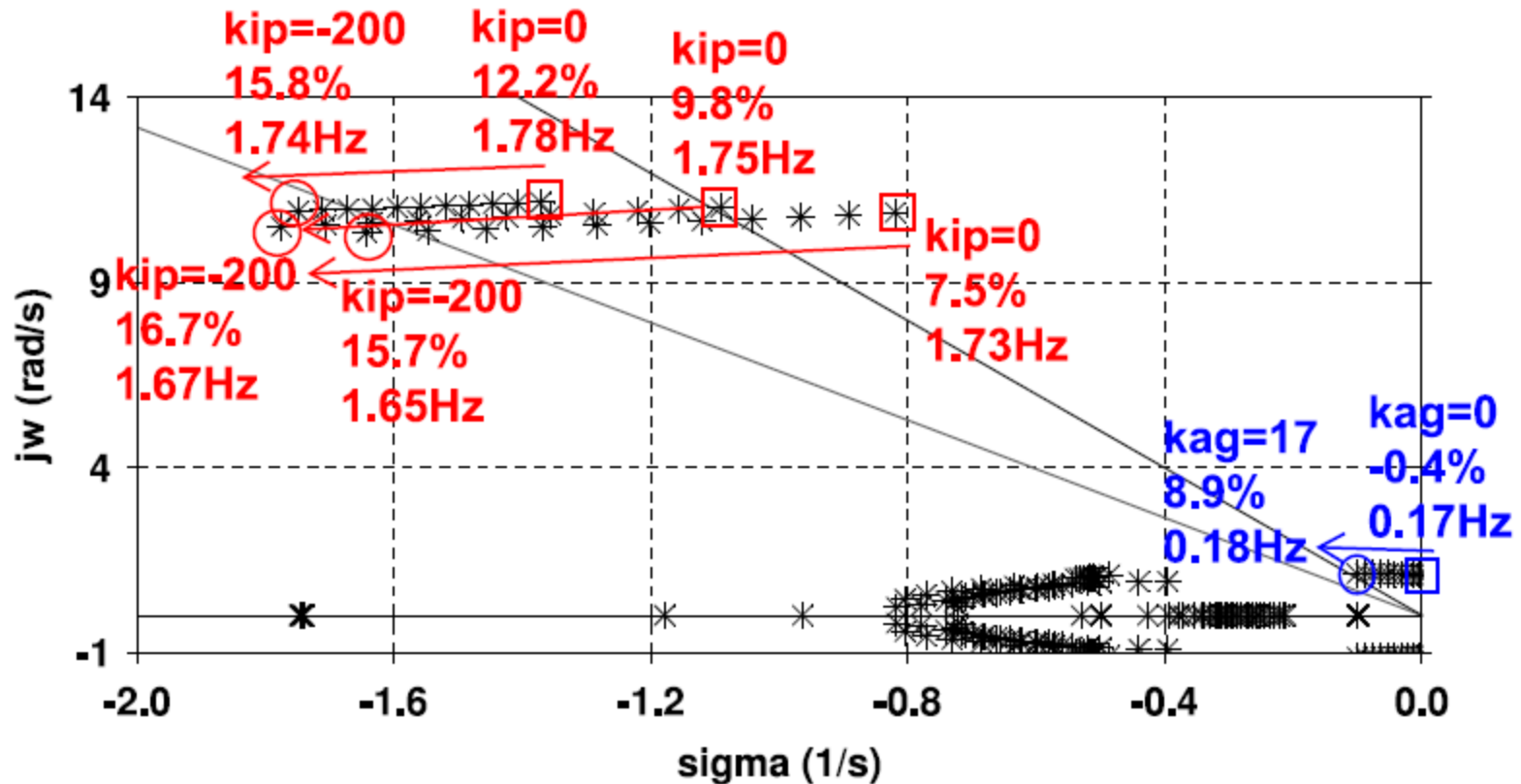


Fig. 25: RL plot for the Unbalanced MPIB Slow-exc system, with the simultaneous variation in the gains of the four 2-channel PSSs. Gain ranges are 0 to 17 for k_{ag} and 0 to -200 for k_{ip} , which vary in steps of 1.7 and -20, respectively.

3. LINEAR SIMULATIONS

Eigenvalue Results for Imbalanced MPIB System

Modes	Standard PSS	2-Channel PSS	Without PSS
Aggregate Mode	$\omega_d = 0.18Hz$ $\zeta = 8.4\%$	$\omega_d = 0.18Hz$ $\zeta = 8.9\%$	$\omega_d = 0.17Hz$ $\zeta = -0.4\%$
Intraplant Mode 1	$\omega_d = 1.78Hz$ $\zeta = 3.8\%$	$\omega_d = 1.65Hz$ $\zeta = 15.7\%$	$\omega_d = 1.73Hz$ $\zeta = 7.5\%$
Intraplant Mode 2	$\omega_d = 1.79Hz$ $\zeta = 6.5\%$	$\omega_d = 1.67Hz$ $\zeta = 16.7\%$	$\omega_d = 1.75Hz$ $\zeta = 9.8\%$
Intraplant Mode 3	$\omega_d = 1.79Hz$ $\zeta = 10.2\%$	$\omega_d = 1.74Hz$ $\zeta = 15.8\%$	$\omega_d = 1.78Hz$ $\zeta = 12.2\%$

3. LINEAR SIMULATIONS

Imbalanced MPIB System with Small Symmetric Disturbance

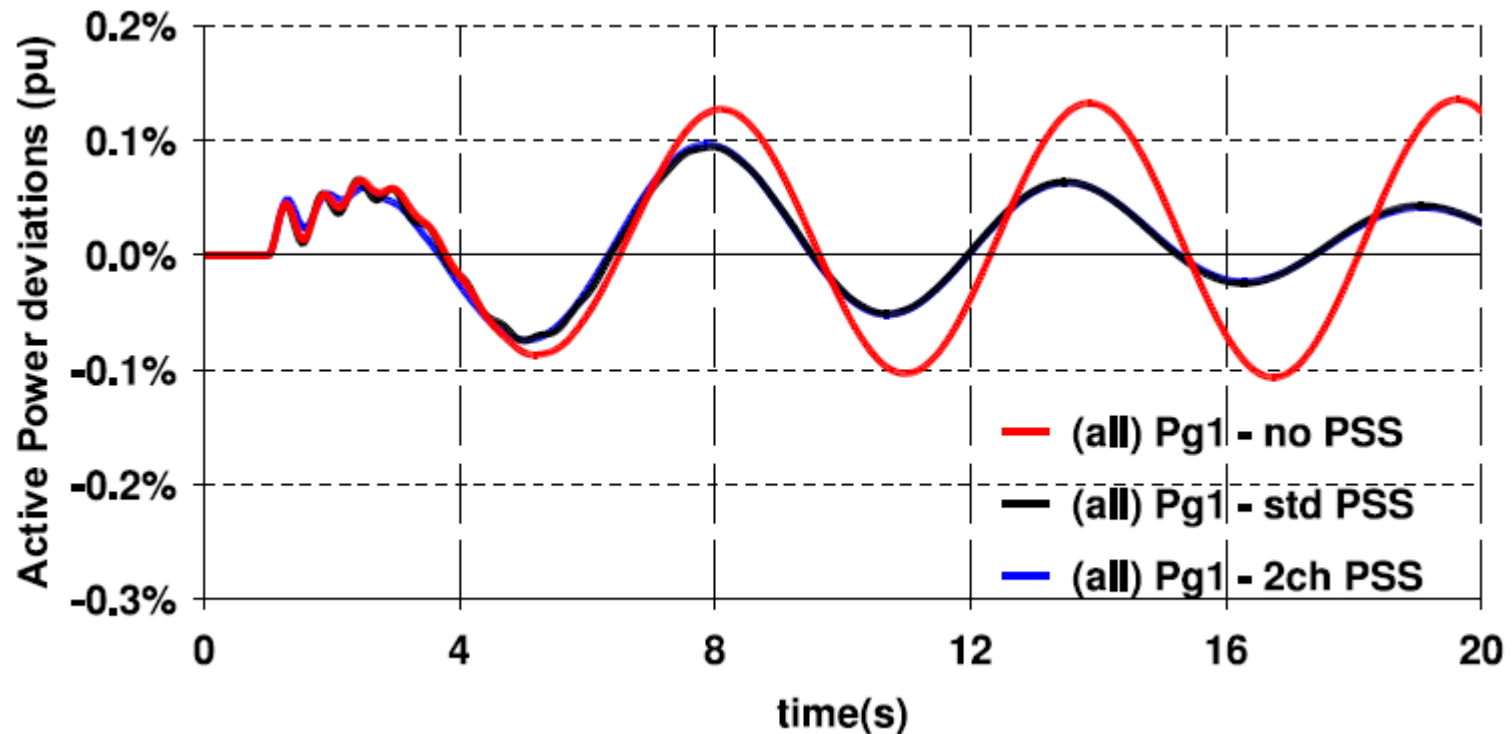


Fig. 26: Unbalanced MPIB Slow-exc system with Small Symmetric disturbance - Active Power responses of one unit.

3. LINEAR SIMULATIONS

Imbalanced MPIB System with Small Asymmetric Disturbance

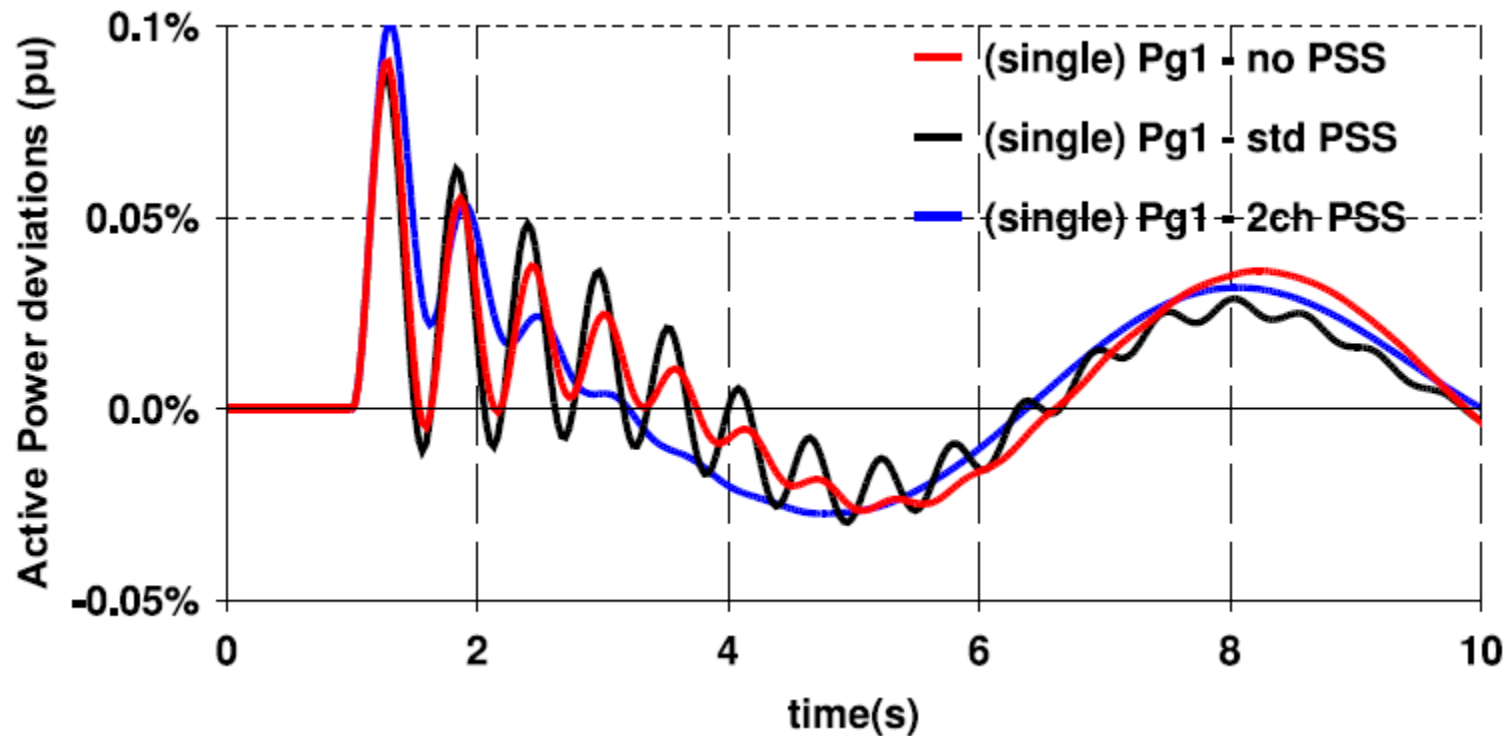


Fig. 27: Unbalanced MPIB Slow-exc system with Small Asymmetric disturbance - Active Power responses of one unit.

4. NONLINEAR (TransStab) SIMULATIONS

MPIB Test System with Slow Response Exciters

Nonlinear Simulation Parameters

- Total simulation time: 20s or 30s, when studying Large Exogenous Faults
- Total simulation time: 6s, when studying Large Internal Faults
- Integration time step: 0.005s
- Fault inception: 1.00s
- Fault duration: 100ms
- Fault at the generator terminals simulated by switching a 800 MVar reactor at the E_1 (generator #1) bus
- Fault at the plant high-side bus simulated by switching a 1080 MVar reactor at the E (high-side) bus

4. NONLINEAR SIMULATIONS

Balanced MPIB System following an External Fault

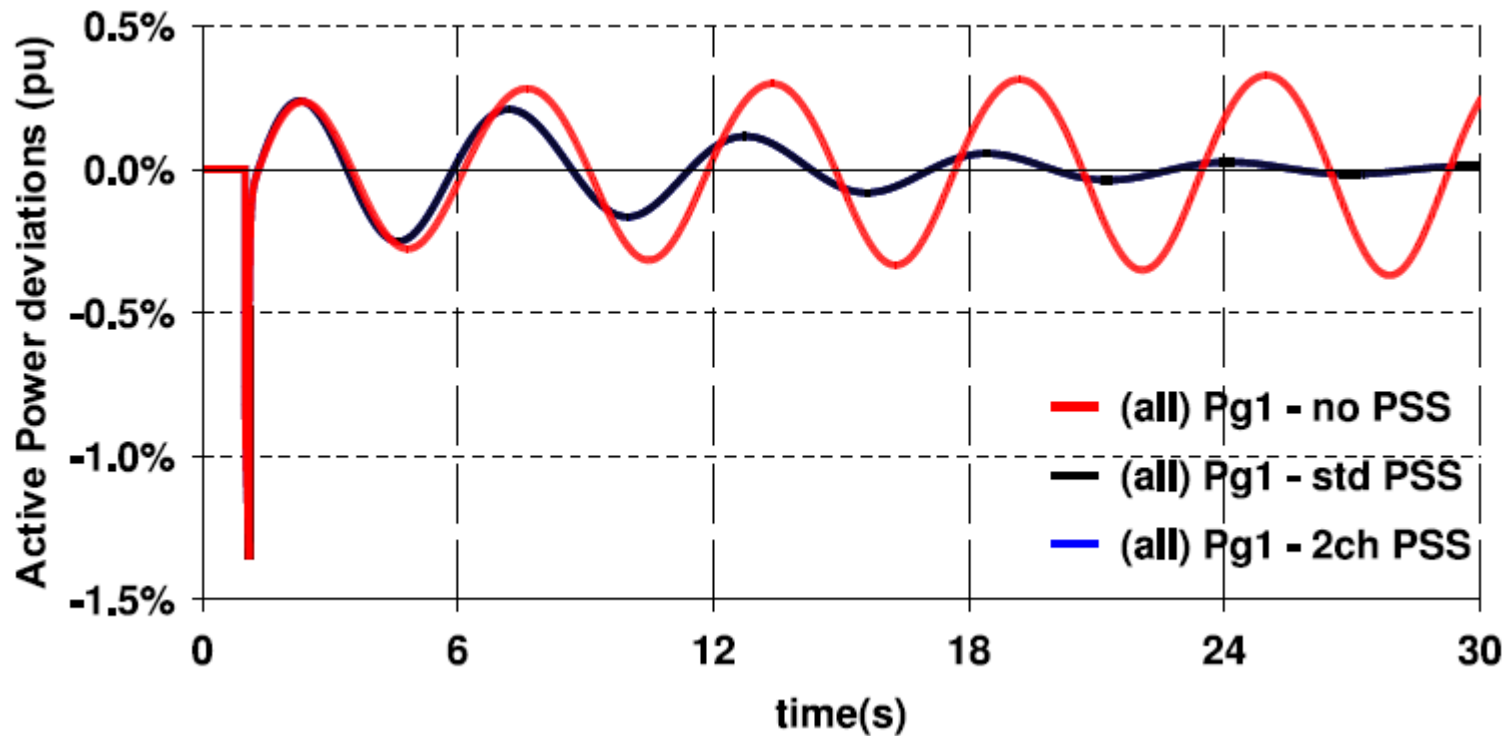


Fig. 28: MPIB Slow-exc system with Large Exogenous Fault
- Active Power responses of unit #1.

4. NONLINEAR SIMULATIONS

Balanced MPIB System Following an External Fault

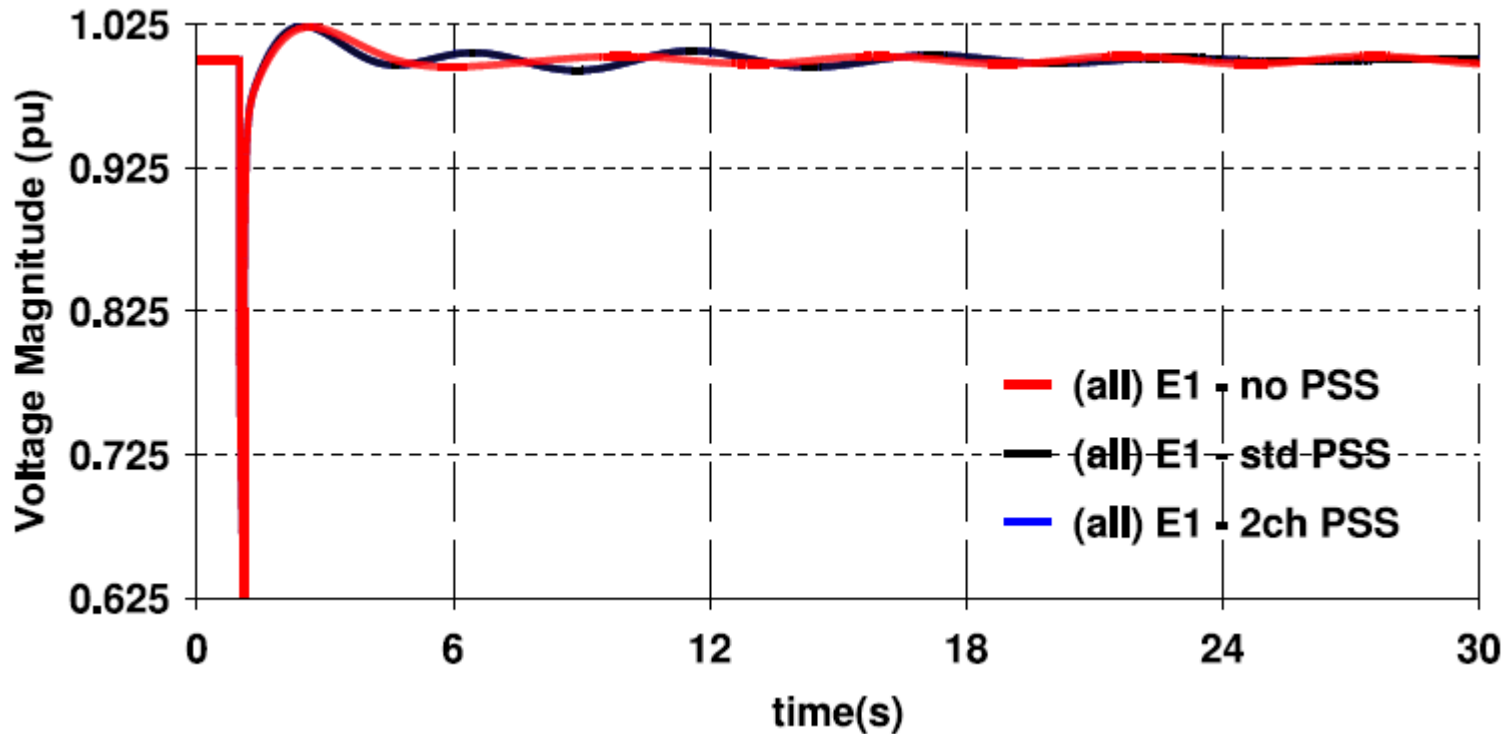


Fig. 29: MPIB Slow-exc system with Large Exogenous Fault
- Terminal Voltage responses of unit #1.

4. NONLINEAR SIMULATIONS

Balanced MPIB System following an External Fault

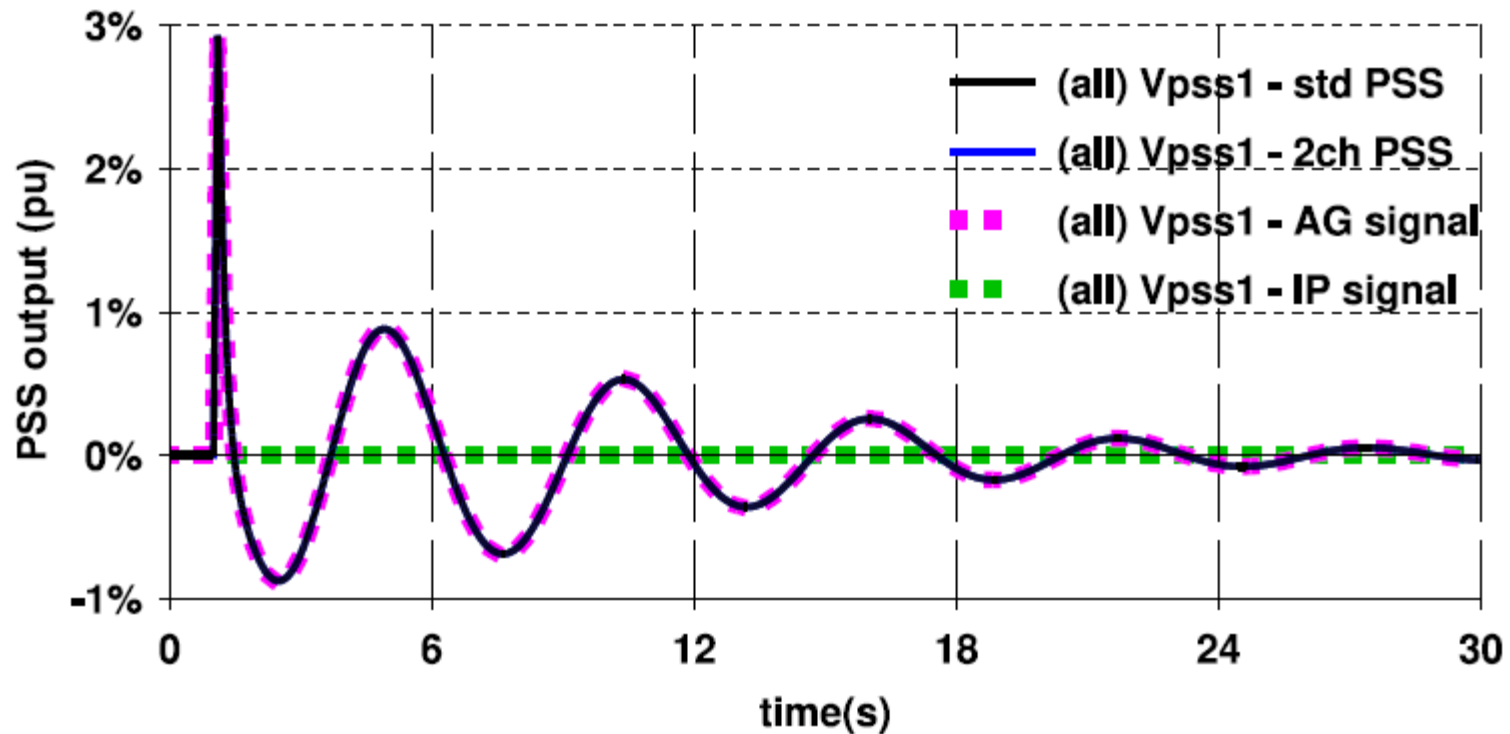


Fig. 30: MPIB Slow-exc system with Large Exogenous Fault - PSS Output responses of unit #1.

4. NONLINEAR SIMULATIONS

Balanced MPIB System following an Internal Fault

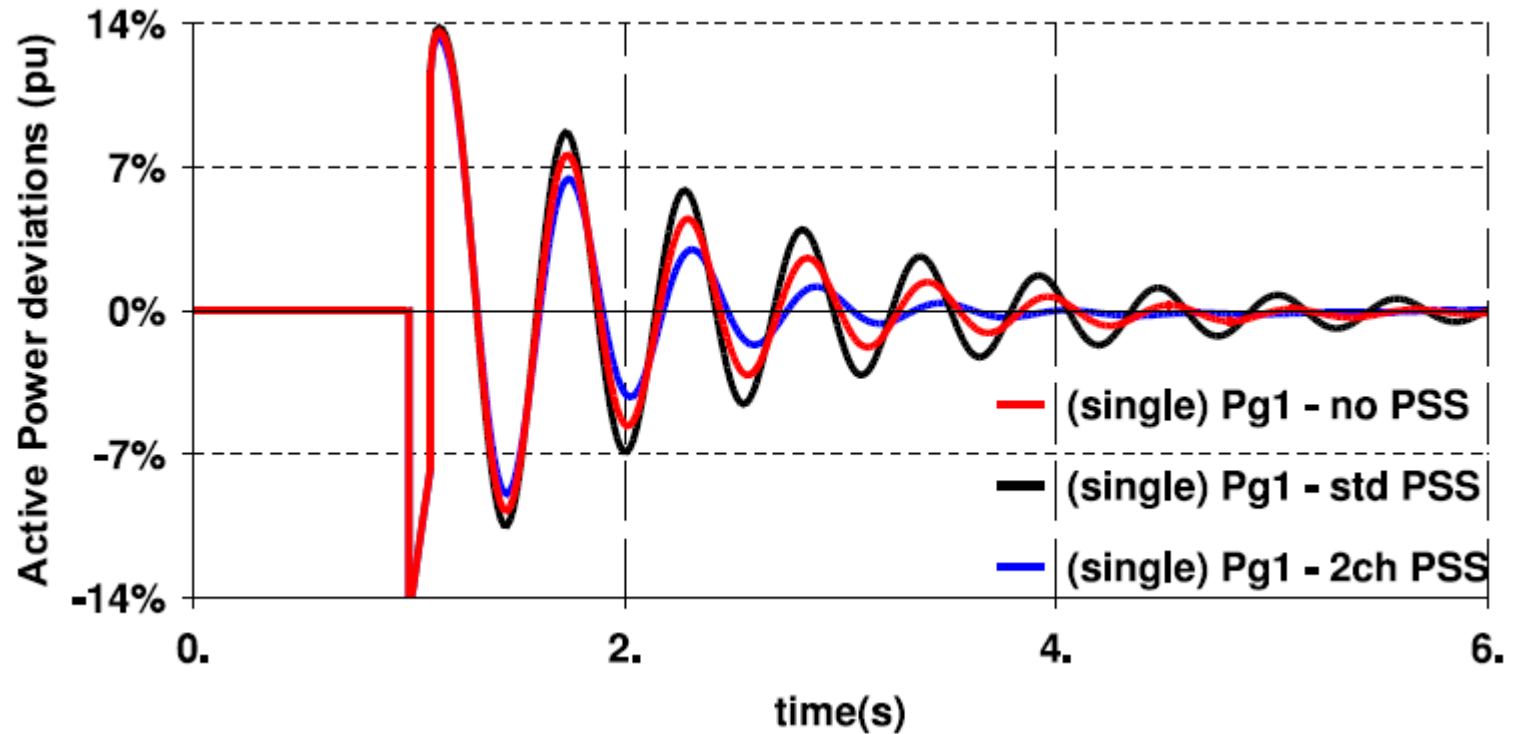


Fig. 31: MPIB Slow-exc system with Large Internal Fault - Active Power responses of unit #1.

4. NONLINEAR SIMULATIONS

Balanced MPIB System following an Internal Fault

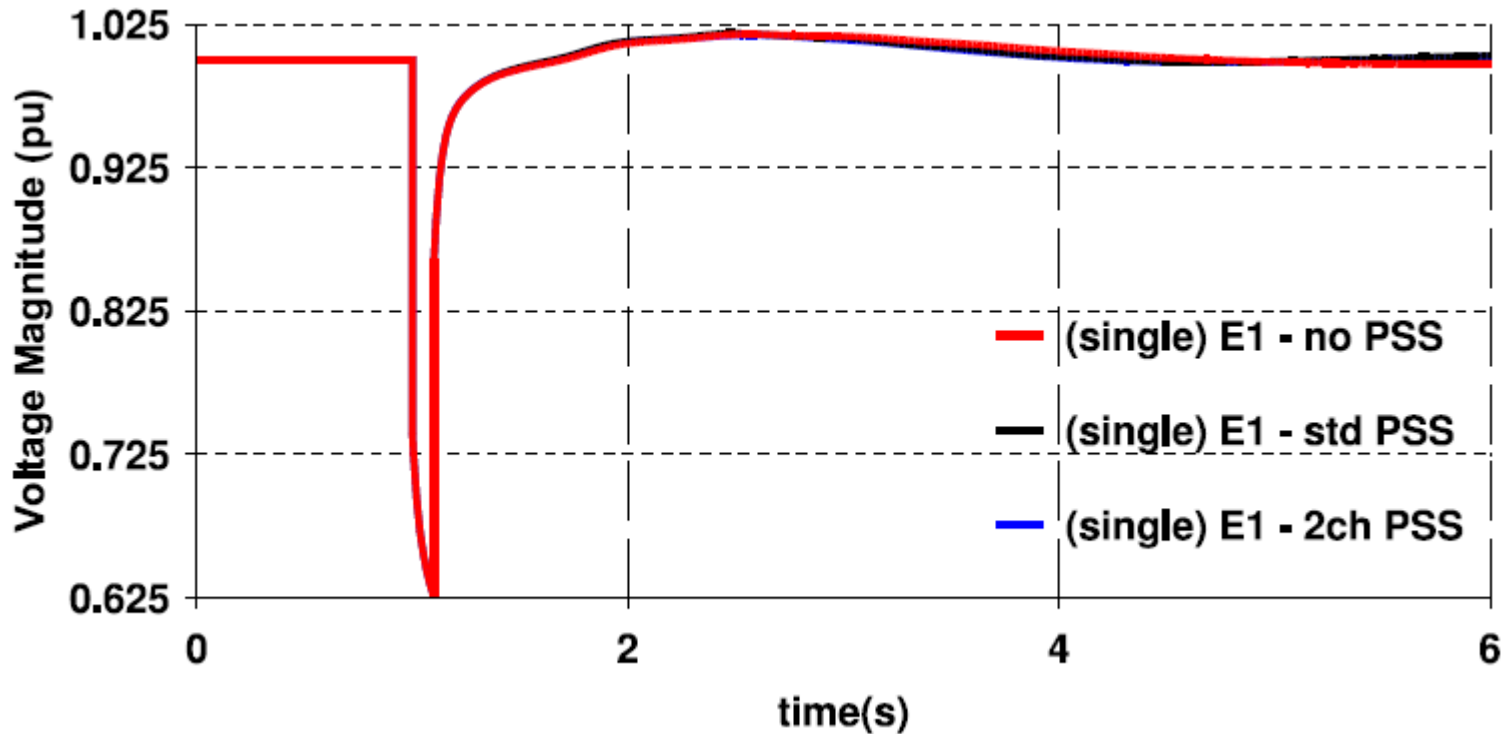


Fig. 32: MPIB Slow-exc system with Large Internal Fault - Terminal Voltage responses of unit #1.

4. NONLINEAR SIMULATIONS

Balanced MPIB System following an Internal Fault

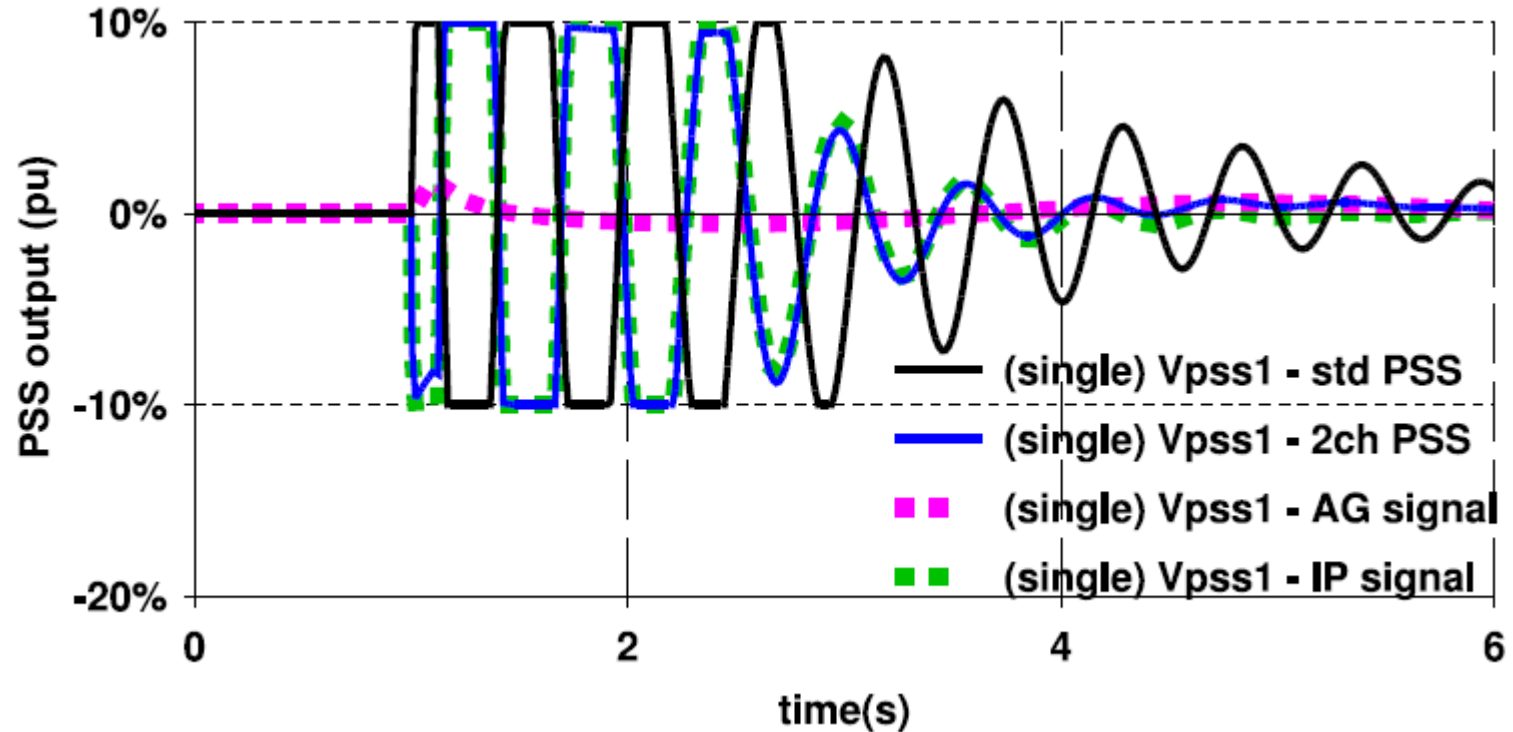


Fig. 33: MPIB Slow-exc system with Large Internal Fault - PSS Output responses of unit #1.

4. NONLINEAR SIMULATIONS

Imbalanced MPIB System following an External Fault (1/2)

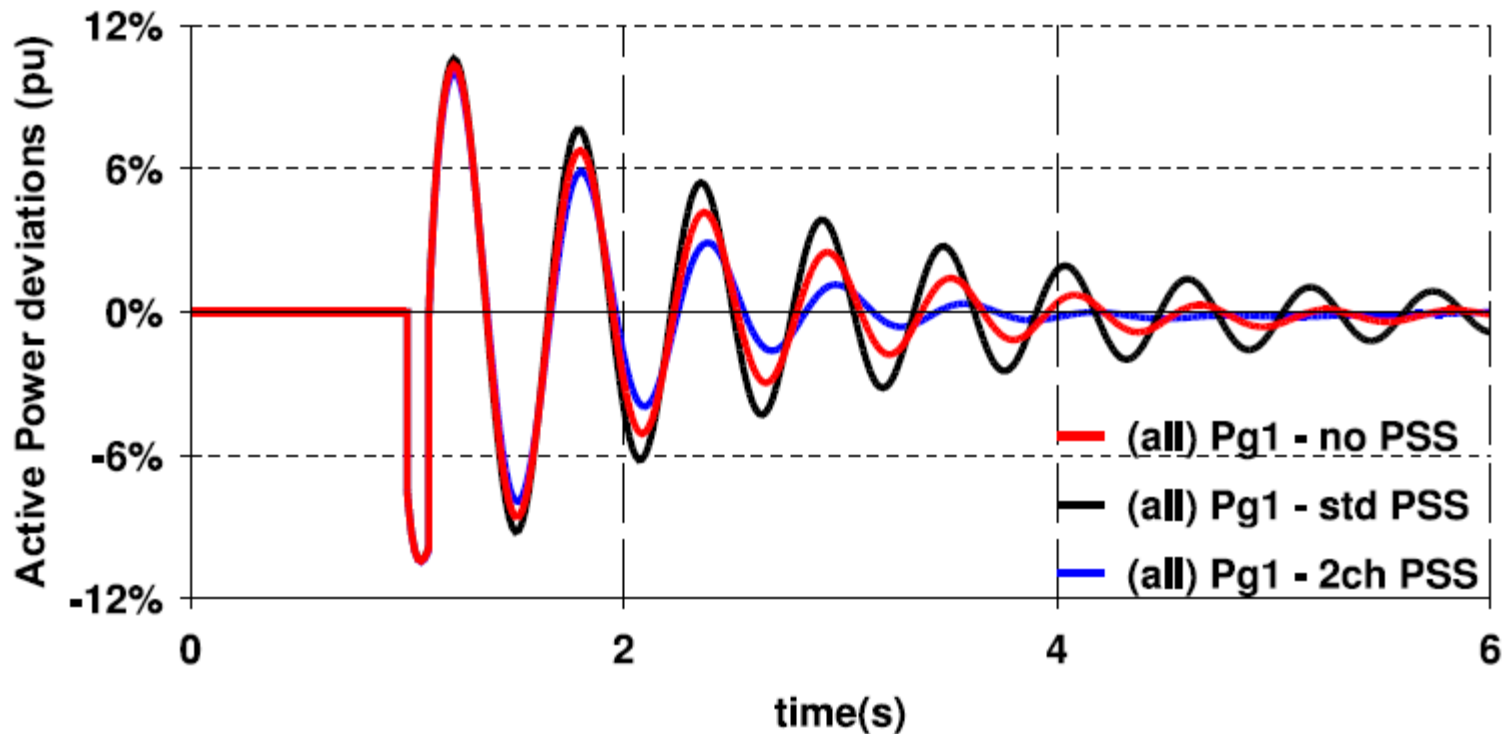


Fig. 34: Unbalanced MPIB Slow-exc system with Large Exogenous Fault - Active Power responses of unit #1. Part I - first 6s of the nonlinear simulation.

4. NONLINEAR SIMULATIONS

Imbalanced MPIB System following an External Fault (2/2)

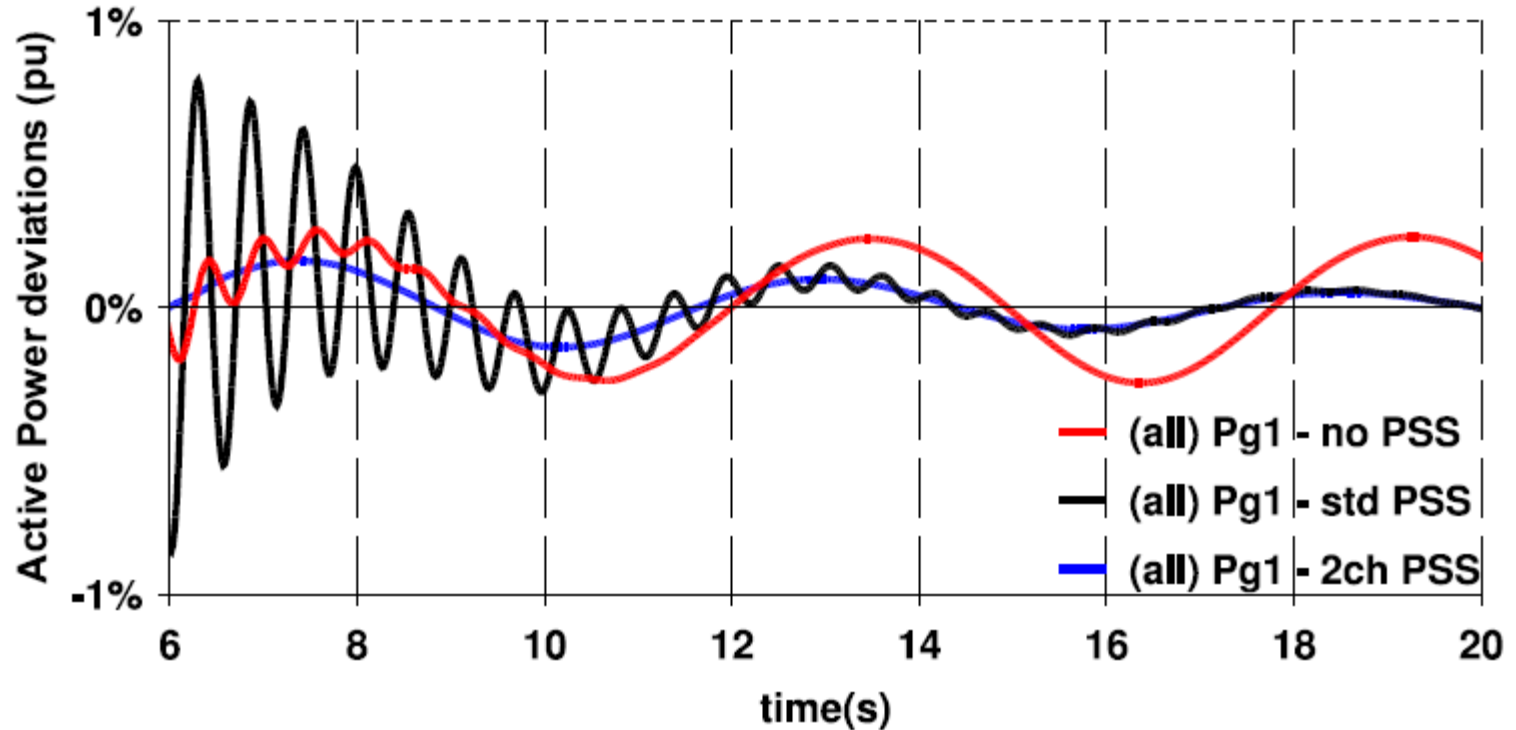


Fig. 35: Unbalanced MPIB Slow-exc system with Large Exogenous Fault - Active Power responses of unit #1. Part II - last 14s of the nonlinear simulation.

5. CONCLUSIONS

Benefits of 2-channel PSS in multigenerator plants

- The intraplant and aggregate components of the V_{pss} signal are orthogonal and maintain the subspace orthogonality that exists in the original system
- Damping ratios for intraplant and aggregate modes can be set as desired by the independent tuning of the two control channels of the 2ch PSS
- Robust damping performance for fairly large levels of plant imbalance
- Helps solving difficult damping control problems in multigenerator plants
- The 2ch PSS solution may prevent discarding rotating exciters when upgrading vintage plants that shall take part in the damping control of interarea modes
- These concepts equally apply to the vibration damping control of light flexible mechanical structures.

7. SIMILARITY TRANSFORMATION

- **A** has a block-symmetric structure
- Similarity transformation with matrix **P** turns the state matrix **A** block-diagonal

$$\mathbf{A} = \begin{bmatrix} \mathbf{a} & \mathbf{b} & \cdots & \mathbf{b} \\ \mathbf{b} & \mathbf{a} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{b} \\ \mathbf{b} & \cdots & \mathbf{b} & \mathbf{a} \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{I}_{m \times m} & \mathbf{I}_{m \times m} & \mathbf{I}_{m \times m} & \cdots & \mathbf{I}_{m \times m} \\ \mathbf{I}_{m \times m} & -\mathbf{I}_{m \times m} & \mathbf{0}_{m \times m} & \cdots & \mathbf{0}_{m \times m} \\ \mathbf{I}_{m \times m} & \mathbf{0}_{m \times m} & -\mathbf{I}_{m \times m} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \mathbf{0}_{m \times m} \\ \mathbf{I}_{m \times m} & \mathbf{0}_{m \times m} & \cdots & \mathbf{0}_{m \times m} & -\mathbf{I}_{m \times m} \end{bmatrix}$$

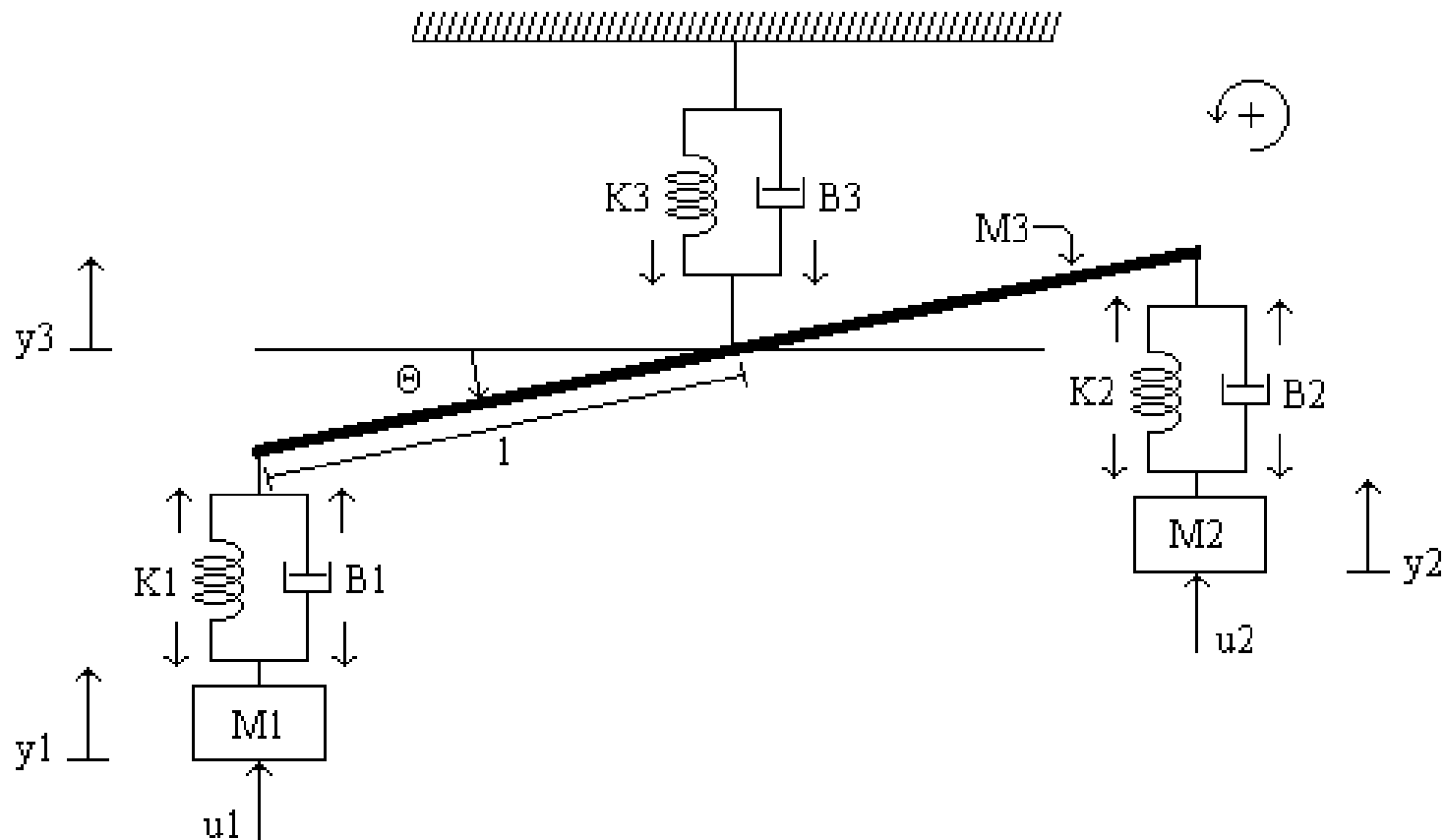
$$\bar{\mathbf{A}} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P}$$

$$\bar{\mathbf{A}} = \begin{bmatrix} \boxed{\mathbf{a} + (n-1)\mathbf{b}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boxed{\mathbf{a} - \mathbf{b}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \boxed{\mathbf{a} - \mathbf{b}} \end{bmatrix}$$

7. MODAL DECOMPOSITION

Mechanical analog

- Spring – Mass System is an analog to the 2-unit Power Plant
 - translational mode ($\theta=0$) is the aggregate mode
 - Rotational mode ($y_3=0$) is the intraplant mode



7. MODAL DECOMPOSITION

Symmetrical Components Analogy

- Impedance of a balanced 3-phase load Z_{bal} :

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \underbrace{\begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}}_{Z_{\text{bal}}} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \mathcal{T} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1\angle 120 & 1\angle -120 \\ 1 & 1\angle -120 & 1\angle 120 \end{bmatrix}$$

- Load is decomposed into its sequence components:

$$Z'_{\text{bal}} = \mathcal{T}^{-1} Z_{\text{bal}} \mathcal{T}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}}_{Z'_{\text{bal}}} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

THANK YOU!