N. Martins Talk - Part 2

Modeling and Simulation of RLC Networks & Modal Equivalents for Transmission Networks Containing Distributed Parameter Lines

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and multiple co-authors*

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Outline of Part II

- Introduction;
- Modeling electrical network components in the formulations Descriptor System (DS) and **Y**(*s*) matrix;
- Distributed parameter transmission line model for Y(s) matrix
- The Sequential MIMO Dominant Pole Algorithm (SMDPA) for computing the dominant poles and residue matrices associated with MIMO TFs of infinite systems;
- Performance of a multi-bus equivalent (MIMO ROM) for a transmission network with distributed parameter lines (poles computed by SMDPA);
- Modeling infinite systems by Linear Matrix Approximations

Introduction to Part II (1/2)

- Modal Analysis
  - Involves the calculation of the system matrix, its poles & zeros and their sensitivities to system parameters;
  - Provides system structural information: mode shapes, participation factors, TF dominant poles, reduced order models;
  - Matrix models are used for the study of different power system phenomena:
    - Eletromechanical transients (Algebraic network modeling, R+jX);
    - Subsynchronous resonance (Lumped R-L-C dynamic network modeling);
    - Harmonic performance;
    - Electromagnetic Transients.

    (High-frequency network modeling, all transmission lines having distributed parameters)

Introduction to Part II (2/2)

- High Frequency Modeling of Electrical Networks
  - 3 formulations: State Space (SS), Descriptor Systems (DS) and $\mathbf{Y}(s)$ matrix;
  - The distributed parameter nature of transmission lines (TL) can be modeled by transcendental functions having infinite poles – Infinite systems;
  - Infinite systems are neatly modeled by the $\mathbf{Y}(s)$ matrix formulation;
  - Finite approximations of infinite systems can be modeled in the SS and DS formulations, where TLs are represented by cascaded RLC circuits;
  - Various NLA methods exist to efficiently compute ROMs for large scale DS models;
  - A main disadvantage of the $\mathbf{Y}(s)$ matrix formulation is the inexistance of robust and efficient algorithms for the computation of the poles and residue matrices of multivariable TFs.

Descriptor Systems (1/5)

- Basic equations:

$$\mathbf{T}\,\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}^T\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)$$

- The components of the system are described by first-order ordinary differential equations and algebraic equations as well;
- The Kirchhoff Law of Currents for each individual node of the network is then added to these equations, to define the connection among the various existing system components;
- The DS model is a generalization of the SS model and leads to a simpler and more efficient computer implementation.

Descriptor Systems (2/5)

- Transfer Functions

TF MIMO $\rightarrow$
$$\mathbf{H}(s) = \mathbf{C}^T (s\,\mathbf{T} - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D}$$

TF SISO $\rightarrow$
$$H(s) = \frac{y(s)}{u(s)} = \mathbf{c}^T (s\,\mathbf{T} - \mathbf{A})^{-1} \mathbf{b} + d$$

- Frequency Response

$$H(j\omega) = \mathbf{c}^T (j\omega\,\mathbf{T} - \mathbf{A})^{-1} \mathbf{b} + d$$

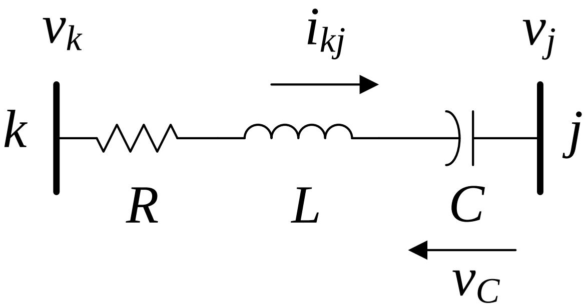
- Time Response (trapezoidal rule of integration)

$$\left(\frac{2}{\Delta t}\mathbf{T} - \mathbf{A}\right)\mathbf{x}(t+\Delta t) = \left(\frac{2}{\Delta t}\mathbf{T} + \mathbf{A}\right)\mathbf{x}(t) + \mathbf{B}\left[\,\mathbf{u}(t) + \mathbf{u}(t+\Delta t)\,\right]$$

$$\mathbf{y}(t+\Delta t) = \mathbf{C}^T \mathbf{x}(t+\Delta t) + \mathbf{D}\,\mathbf{u}(t+\Delta t)$$

Descriptor Systems (3/5)

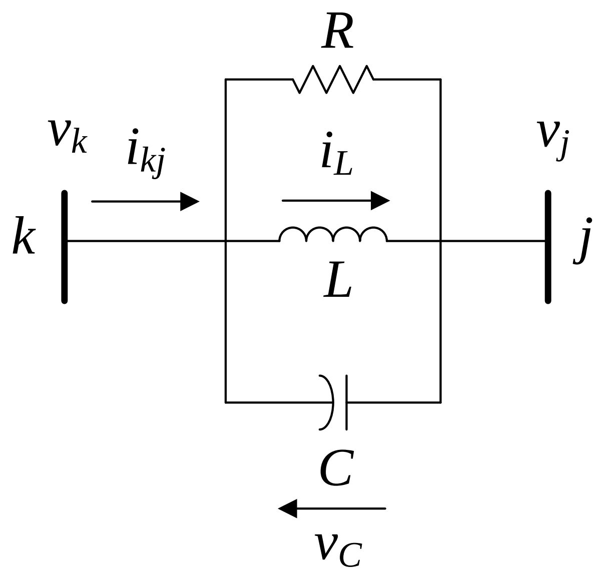
RLC Series



$$L\frac{di_{kj}}{dt} = -v_C - R\,i_{kj} + v_k - v_j$$

$$C\frac{dv_C}{dt} = i_{kj}$$

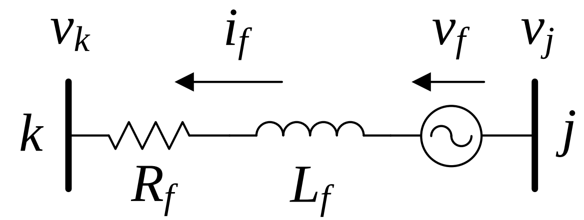
RLC Parallel



$$C\frac{dv_C}{dt} = -i_L - \frac{1}{R}v_C + i_{kj}$$

$$L\frac{di_L}{dt} = v_C \qquad v_k - v_j - v_C = 0$$

Voltage Source



$$L_f\frac{di_f}{dt} = -R_f\,i_f - v_k + v_j + v_f$$

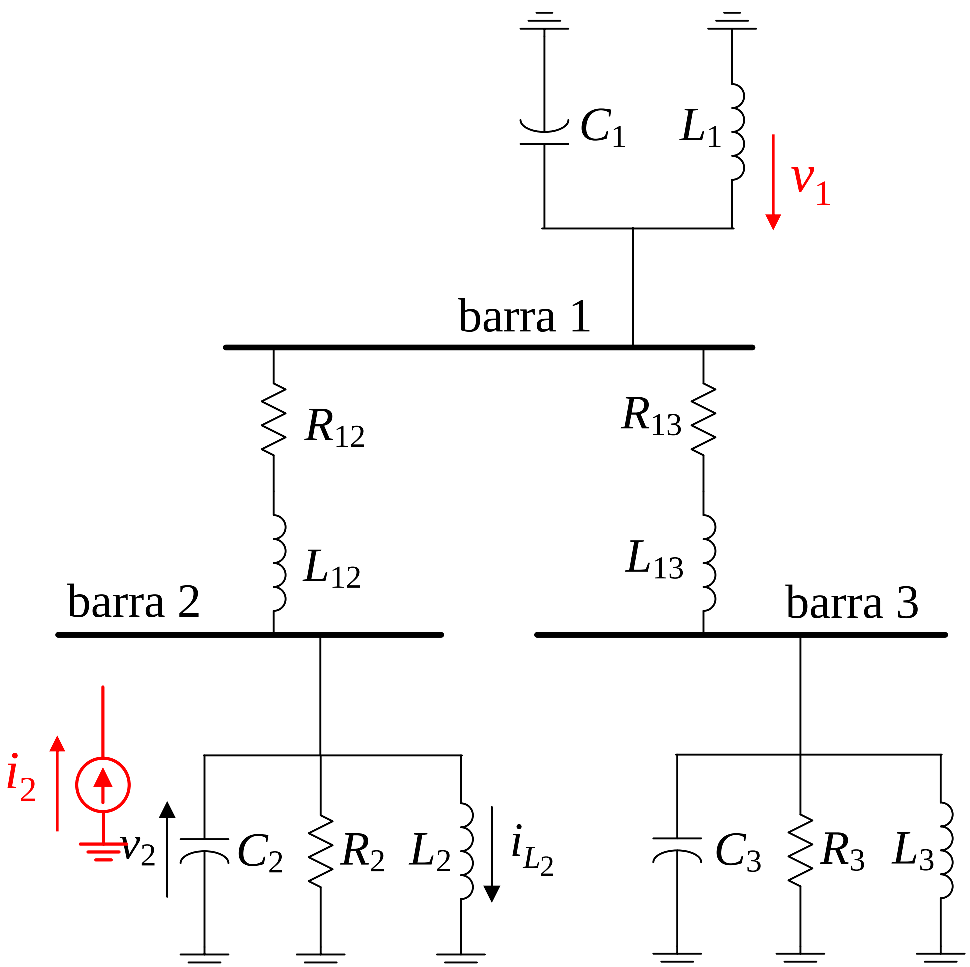
Kirchhoff Current Law

$$\text{Node } k \longrightarrow \sum_{m\in\Omega} i_{mk} = 0$$

$$\Omega \longrightarrow \text{Nodes connected to } k$$

Descriptor Systems (4/5)

Matlab script vs PSCAD Validation



Parameters for 3-bus system

Ind. (mH)		Res. (Ω)		Cap. (μF)	
L_1	8.0	R_2	80.0	C_1	23.9
L_2	424.0	R_3	133.0	C_2	8.0
L_3	531.0	R_{12}	0.46	C_3	11.9
L_{12}	9.7	R_{13}	0.55		
L_{13}	11.9				

Input $\longrightarrow i_2 = 1$ pu

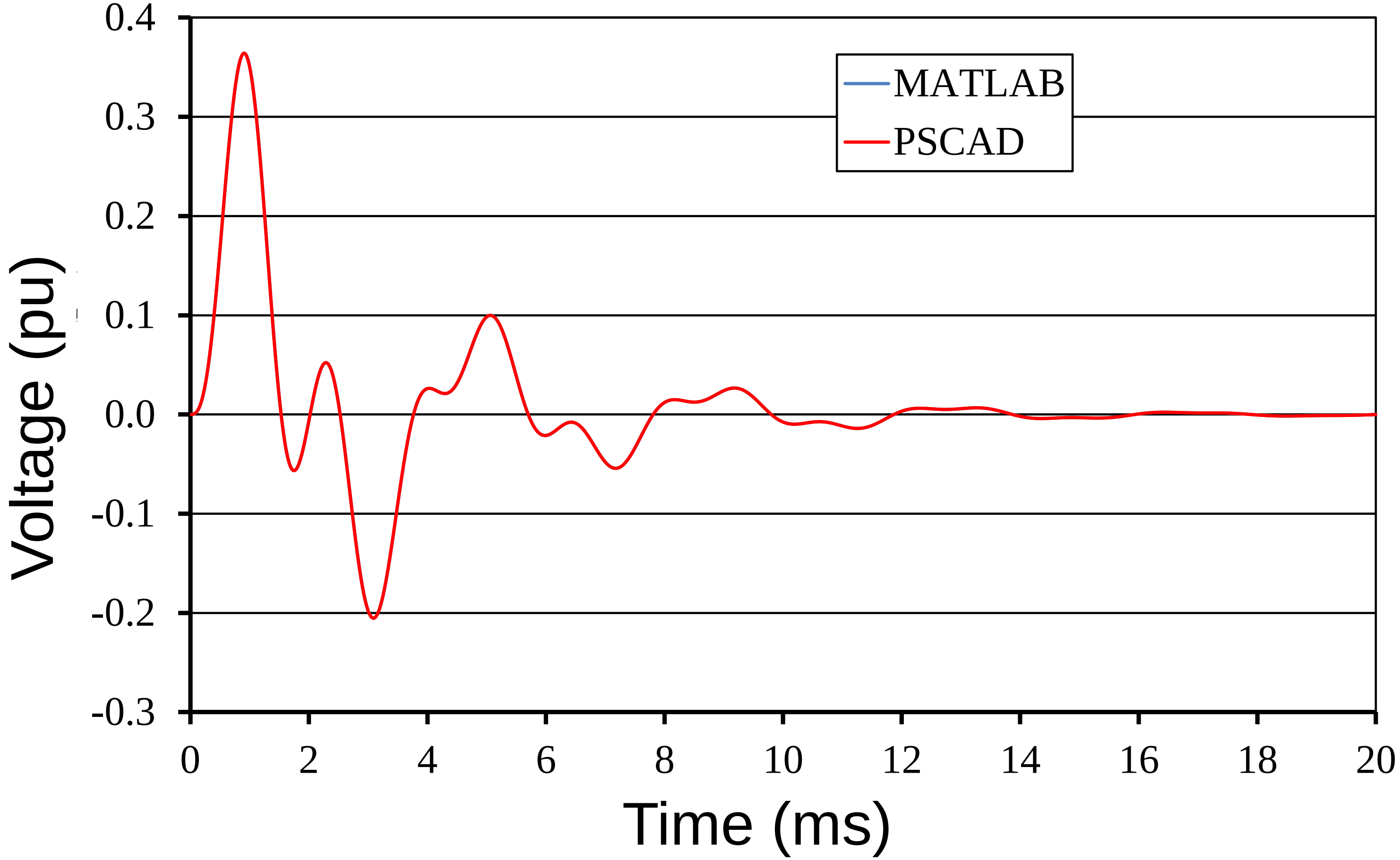
Output $\longrightarrow v_1$ (pu)

Nominal Frequency: 50 Hz
Nominal Voltage: 20 kV
MVA base: 10 MVA

Descriptor Systems (5/5)

Matlab script vs PSCAD Validation

Voltage at Bus # 1 following a step in the current injected in Bus 2



$$C_3 \frac{dv_{C_3}}{dt} = -\,i_{L_3} - \frac{1}{R_3} v_{C_3} + i_{30} \qquad (7)$$

$$0 = -i_{20} + i_{12} + i_2 \qquad (13)$$

$$
\begin{array}{l}
\\ \\ \\ \\ \\ \\ (7)\rightarrow \\ \\ \\ \\ \\ \\ (13)\rightarrow \\ \\
\end{array}
\left[\begin{array}{ccccccccccc|ccc}
C_1 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & L_2 & & & & & & & & & & & \\
 & & & C_2 & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & L_3 & & & & & & & & \\
 & & & & & & C_3 & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & L_{12} & & & & & \\
 & & & & & & & & & L_{13} & & & & \\
 & & & & & & & & & & L_f & & & \\
\hline
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & &
\end{array}\right]
\frac{d}{dt}
\left[\begin{array}{c}
v_{C_1} \\ i_{10} \\ i_{L_2} \\ v_{C_2} \\ i_{20} \\ i_{L_3} \\ v_{C_3} \\ i_{30} \\ i_{12} \\ i_{13} \\ i_f \\ \hline v_1 \\ v_2 \\ v_3
\end{array}\right] =
$$

$$
\begin{array}{l}
\\ \\ \\ \\ \\ \\ (7)\rightarrow \\ \\ \\ \\ \\ \\ (13)\rightarrow \\ \\
\end{array}
\left[\begin{array}{ccccccccccc|ccc}
 & 1 & & & & & & & & & & & & \\
-1 & & & & & & & & & & & 1 & & \\
 & & & 1 & & & & & & & & & & \\
 & & -1 & -1/R_2 & 1 & & & & & & & & & \\
 & & & -1 & & & & & & & & & 1 & \\
 & & & & & & 1 & & & & & & & \\
 & & & & & -1 & -1/R_3 & 1 & & & & & & \\
 & & & & & & -1 & & & & & & & 1 \\
 & & & & & & & & -R_{12} & & & 1 & -1 & \\
 & & & & & & & & & -R_{13} & & 1 & & -1 \\
 & & & & & & & & & & -R_f & -1 & & \\
\hline
 & -1 & & & & & & & -1 & -1 & 1 & & & \\
 & & & & -1 & & & & 1 & & & & & \\
 & & & & & & & -1 & & 1 & & & &
\end{array}\right]
\left[\begin{array}{c}
v_{C_1} \\ i_{10} \\ i_{L_2} \\ v_{C_2} \\ i_{20} \\ i_{L_3} \\ v_{C_3} \\ i_{30} \\ i_{12} \\ i_{13} \\ i_f \\ \hline v_1 \\ v_2 \\ v_3
\end{array}\right]
+
\left[\begin{array}{ccc}
 & & \\
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1 & & \\
\hline
 & & \\
 & 1 & \\
 & & 1
\end{array}\right]
\left[\begin{array}{c}
v_f \\ i_2 \\ i_3
\end{array}\right]
$$

Descriptor System Matrices (2/2)

- Let us consider the nodal voltages as the output variables:

$$
\begin{bmatrix} v_1 \\ \hdashline v_2 \\ \hdashline v_3 \end{bmatrix}
=
\left[\begin{array}{c:c:c:c:c:c:c:c:c:c:c|c:c:c}
 & & & & & & & & & & & 1 & & \\ \hdashline
 & & & & & & & & & & & & 1 & \\ \hdashline
 & & & & & & & & & & & & & 1
\end{array}\right]
\begin{bmatrix} v_{C_1} \\ \hdashline i_{10} \\ \hdashline i_{L_2} \\ \hdashline v_{C_2} \\ \hdashline i_{20} \\ \hdashline i_{L_3} \\ \hdashline v_{C_3} \\ \hdashline i_{30} \\ \hdashline i_{12} \\ \hdashline i_{13} \\ \hdashline i_f \\ \hline v_1 \\ \hdashline v_2 \\ \hdashline v_3 \end{bmatrix}
+
\left[\begin{array}{c:c:c}
 & & \\ \hdashline
 & & \\ \hdashline
 & &
\end{array}\right]
\begin{bmatrix} v_f \\ \hdashline i_2 \\ \hdashline i_3 \end{bmatrix}
$$

Y(s) Matrix Formulation

- Basic equations:

$$\mathbf{Y}(s)\,\mathbf{x}(s) = \mathbf{B}\,\mathbf{u}(s)$$

$$\mathbf{y}(s) = \mathbf{C}^T\mathbf{x}(s) + \mathbf{D}\,\mathbf{u}(s)$$

- Elements
  - Diagonal y_{ii}: Summation of the all elementary admittances connected to node i;
  - Off-diagonal y_{ij}: negative value of summation of all elementary admittances connected between nodes i and j;
- SS and DS formulations are particular cases of Y(s): Y(s) = (sT – A)
- Voltage sources are modeled by additional equations;
- The derivative of $\mathbf{Y}(s)$ with respect to s, for the computation of the system poles, is automatically built by coding simple rules that are similar to those used for building $\mathbf{Y}(s)$.

Y(s) Matrix Formulation

- Transfer Function

MIMO TF $\rightarrow$ $$\mathbf{H}(s) = \mathbf{C}^T \mathbf{Y}(s)^{-1} \mathbf{B} + \mathbf{D}$$

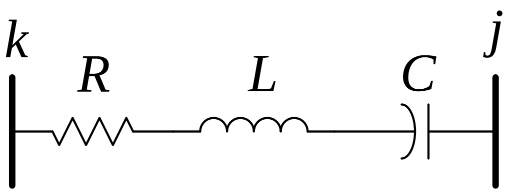
SISO TF $\rightarrow$ $$H(s) = \frac{y(s)}{u(s)} = \mathbf{c}^T \mathbf{Y}(s)^{-1} \mathbf{b} + d$$

- Frequency Response

$$H(j\omega) = \mathbf{c}^T \mathbf{Y}(j\omega)^{-1} \mathbf{b} + d$$

Y(s) Matrix Formulation – Basic Elements

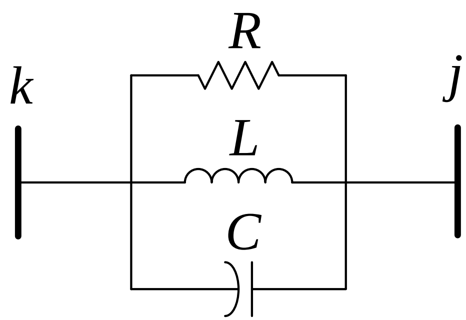
Series RLC



$$y_{series} = \frac{1}{R + s\,L + \dfrac{1}{s\,C}}$$

$$\frac{dy_{series}}{ds} = \frac{-L + \dfrac{1}{s^2 C}}{\left(R + s\,L + \dfrac{1}{s\,C}\right)^2}$$

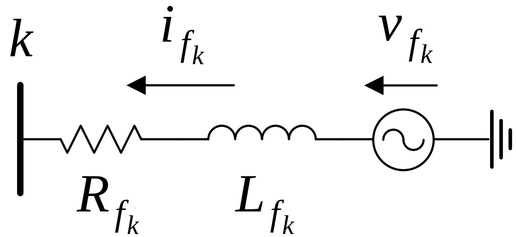
Parallel RLC



$$y_{parallel} = \frac{1}{R} + \frac{1}{sL} + sC$$

$$\frac{dy_{parallel}}{ds} = C - \frac{1}{s^2 L}$$

Voltage Source



$$\sum_{j=1}^{n} y_{kj}\, v_j - i_{f_k} = 0$$

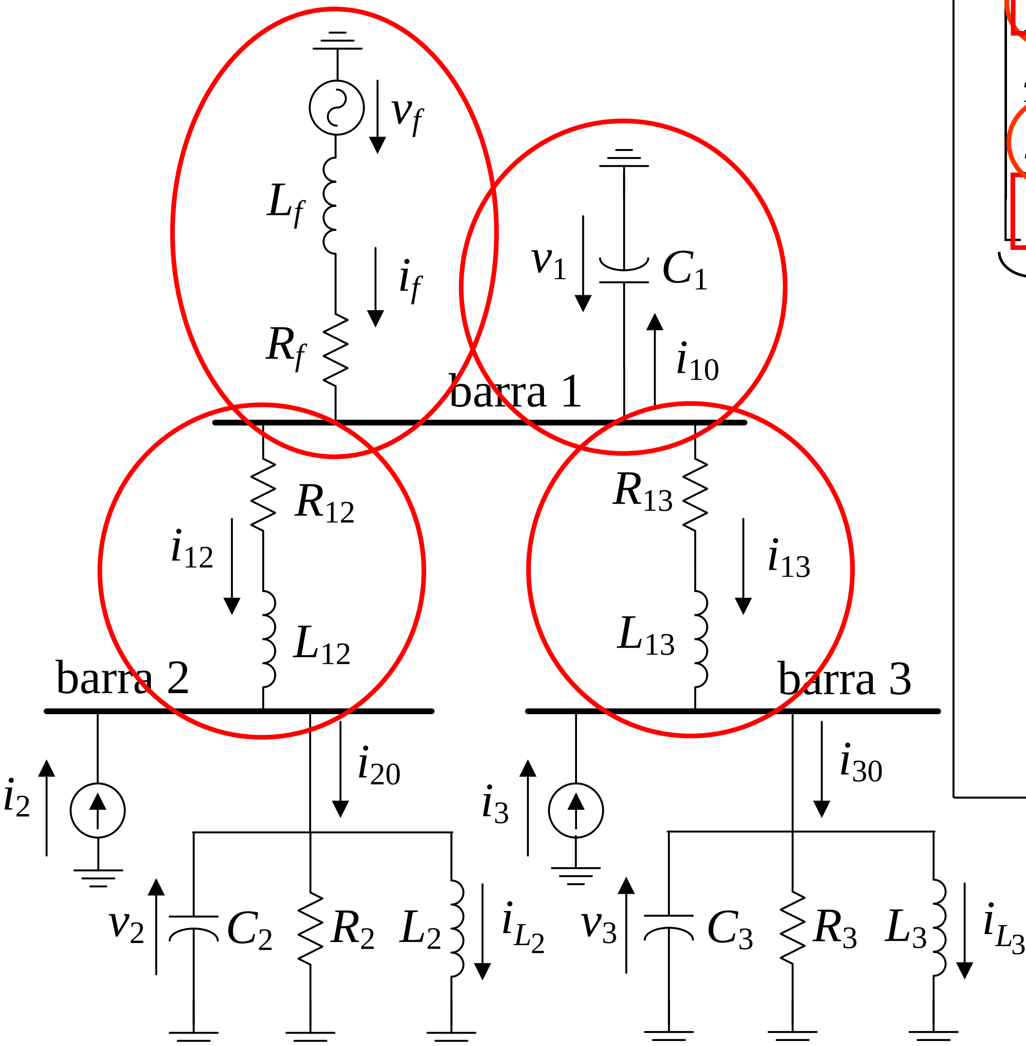
$$v_k + z_{f_k}\, i_{f_k} = v_{f_k}$$

where:

$$z_{f_k} = R_{f_k} + s\,L_{f_k}$$

$$\frac{dz_{f_k}}{ds} = L_{f_k}$$

$\mathbf{Y}(s)$ Matrix – 3-bus System Equations



$$\underbrace{\begin{bmatrix} y_{11} & y_{12} & y_{13} & -1 \\ y_{21} & y_{22} & 0 & 0 \\ y_{31} & 0 & y_{33} & 0 \\ 1 & 0 & 0 & z_f \end{bmatrix}}_{\mathbf{Y}(s)} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_f \end{bmatrix}$$

$$y_{11} = s\,C_1 + \left(\frac{1}{R_{12} + s\,L_{12}}\right) + \left(\frac{1}{R_{13} + s\,L_{13}}\right)$$

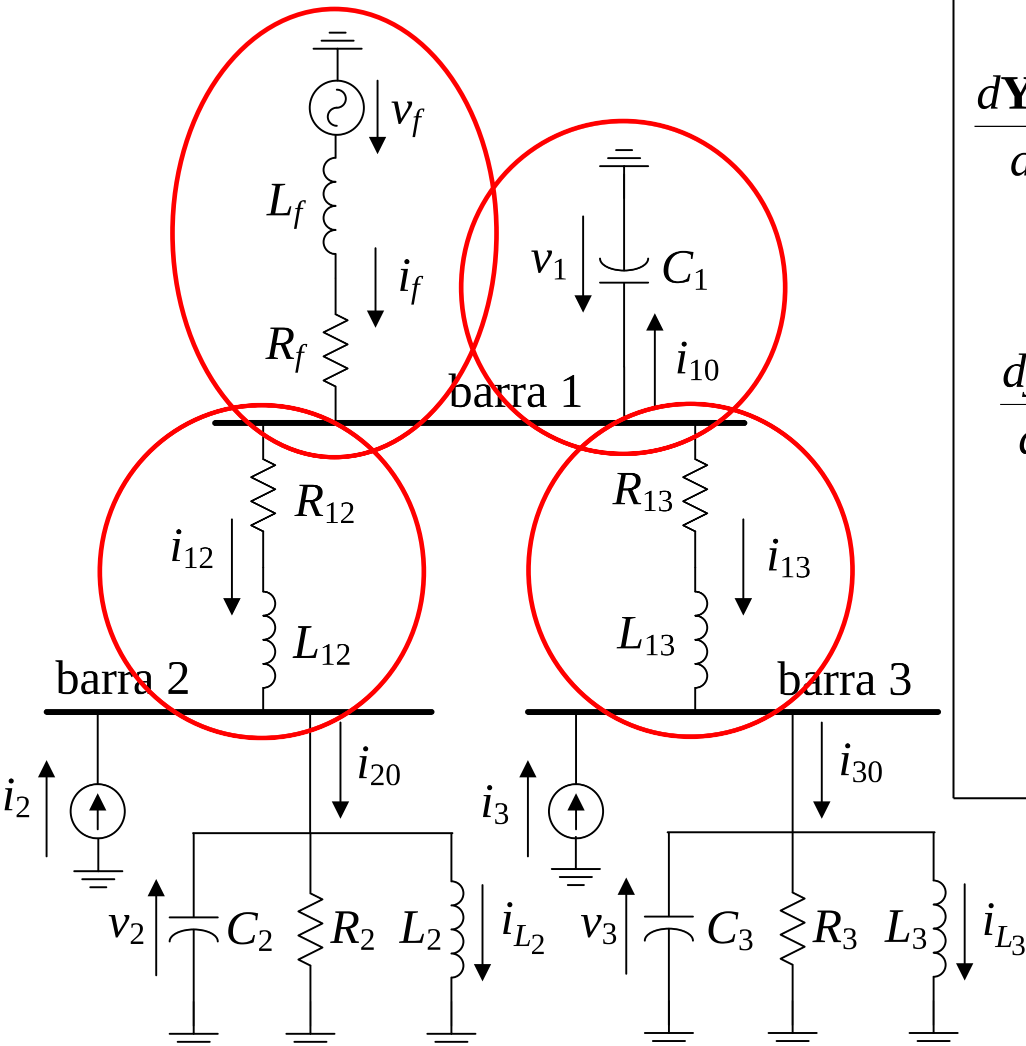
$$y_{13} = y_{31} = -\left(\frac{1}{R_{13} + s\,L_{13}}\right)$$

Voltage Source

LCK $\quad y_{11}\,v_1 + y_{12}\,v_2 + y_{13}\,v_3 - i_f = 0$

LTK $\quad v_1 + z_f\,i_f = v_f$

$\mathbf{Y}(s)$ Matrix – 3-bus System Equations



$$\frac{d\mathbf{Y}(s)}{ds} = \begin{bmatrix} dy_{11}/ds & dy_{12}/ds & dy_{13}/ds & 0 \\ dy_{21}/ds & dy_{22}/ds & 0 & 0 \\ dy_{31}/ds & 0 & dy_{33}/ds & 0 \\ 0 & 0 & 0 & dz_f/ds \end{bmatrix}$$

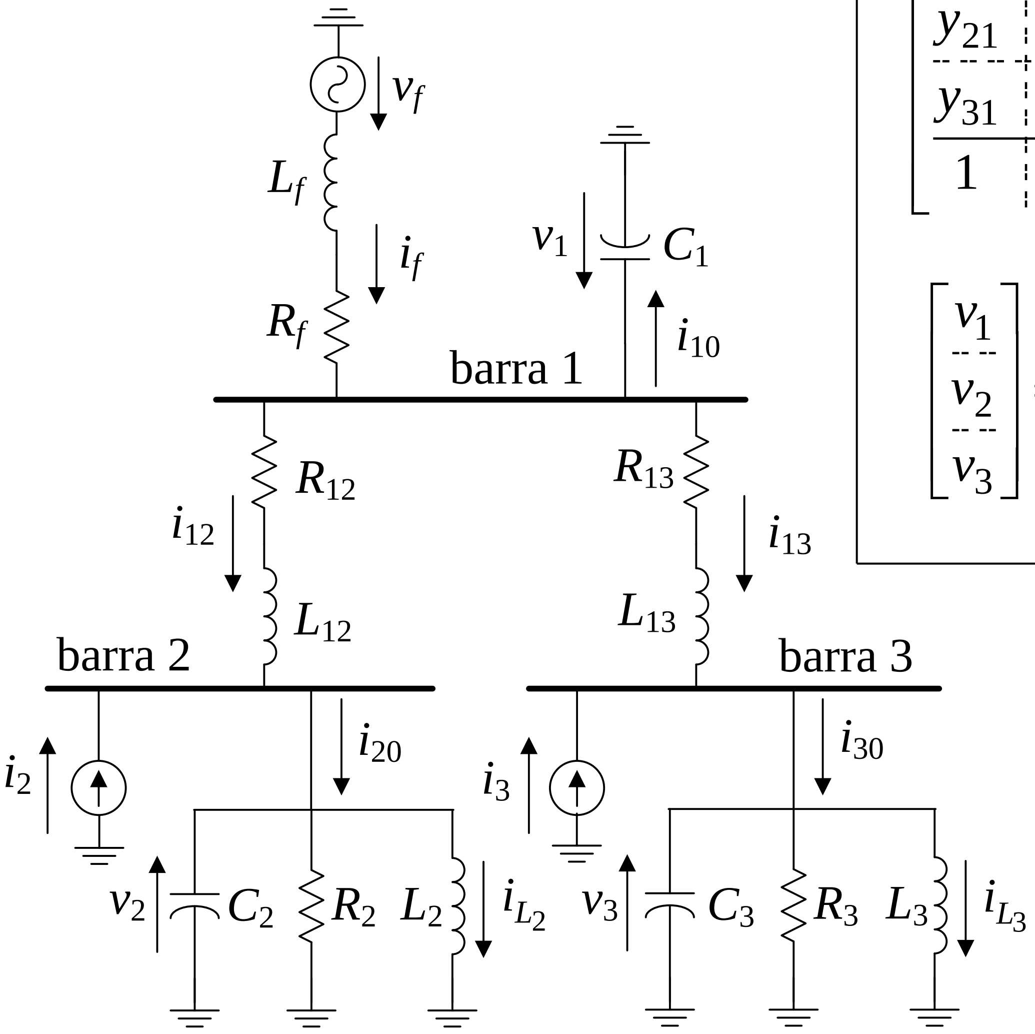
$$\frac{dy_{11}}{ds} = C_1 + \left[\frac{-L_{12}}{(R_{12} + s\,L_{12})^2} \right] + \left[\frac{-L_{13}}{(R_{13} + s\,L_{13})^2} \right]$$

$$\frac{dy_{13}}{ds} = \frac{dy_{31}}{ds} = \frac{L_{13}}{(R_{13} + s\,L_{13})^2}$$

Fonte de Tensão

$$\frac{dz_f}{ds} = \frac{d(R_f + s\,L_f)}{ds} = L_f$$

$\mathbf{Y}(s)$ Matrix – 3-bus System Equations



$$\left[\begin{array}{ccc|c} y_{11} & y_{12} & y_{13} & -1 \\ y_{21} & y_{22} & 0 & 0 \\ y_{31} & 0 & y_{33} & 0 \\ \hline 1 & 0 & 0 & z_f \end{array}\right]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \hline i_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \\ \hline v_f \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \hline i_f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \\ \hline v_f \end{bmatrix}$$

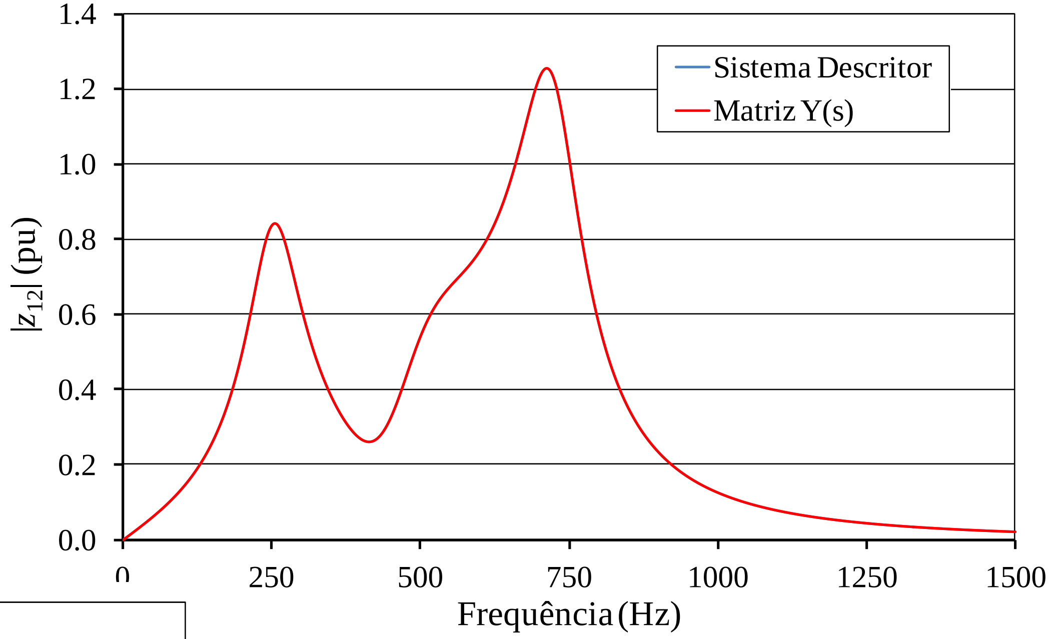
A compact description

$$\mathbf{Y}(s)\,\mathbf{x}(s) = \mathbf{B}\,\mathbf{u}(s)$$

$$\mathbf{y}(s) = \mathbf{C}\,\mathbf{x}(s) + \mathbf{D}\,\mathbf{u}(s)$$

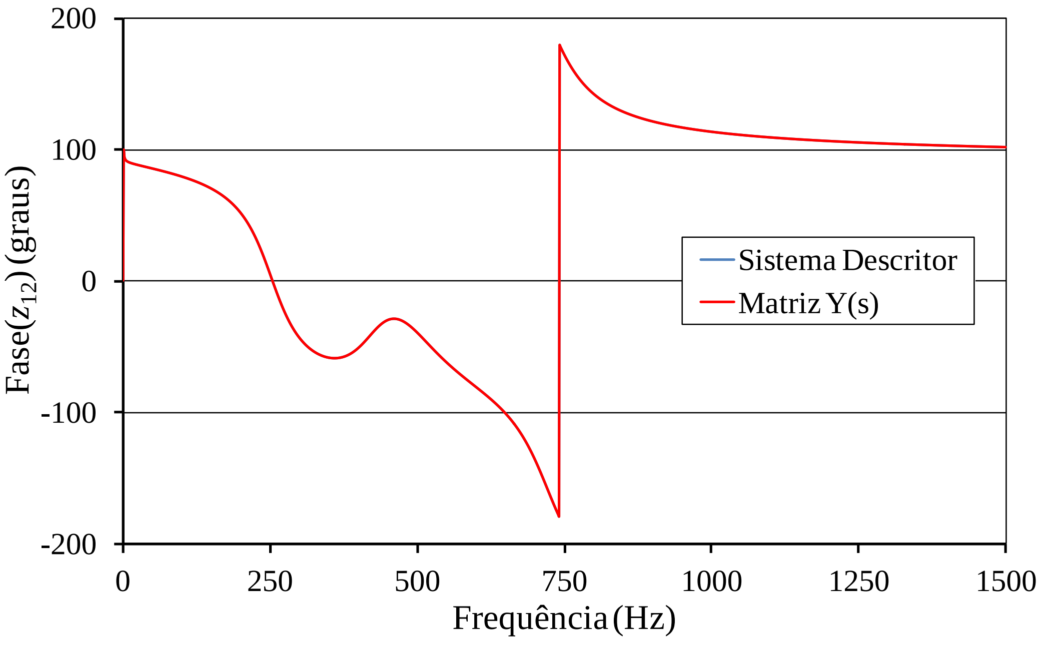
MIMO System

Y(s) matrix- Freq. response results for 3-bus system

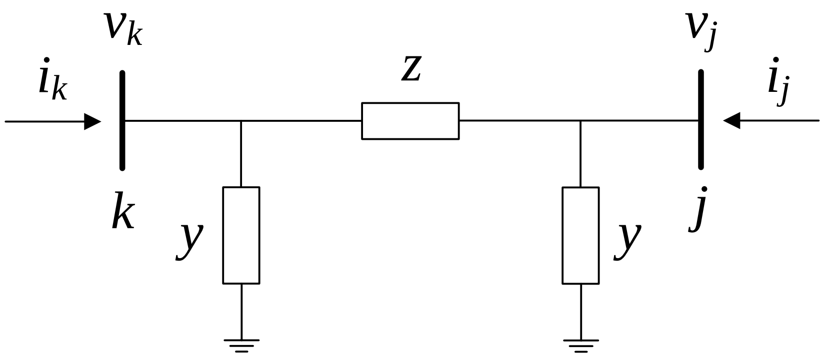


DS compared to Y(*s*) Matrix:

Transfer Impedance between buses 1 & 2



$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line



$$z = z_c \sinh(\gamma\, l) \;\Rightarrow\; \frac{1}{z} = y_c \operatorname{csch}(\gamma\, l)$$

$$y = y_c \tanh\left(\frac{\gamma\, l}{2}\right) = y_c \left[\coth(\gamma\, l) - \operatorname{csch}(\gamma\, l)\right]$$

$$y_c = \sqrt{\frac{Y(s)}{Z(s)}} \qquad z_c = \frac{1}{y_c} \qquad \gamma = \sqrt{Z(s)\,Y(s)}$$

$$\begin{bmatrix} i_k \\ i_j \end{bmatrix} = \begin{bmatrix} y + \frac{1}{z} & -\frac{1}{z} \\ -\frac{1}{z} & y + \frac{1}{z} \end{bmatrix} \begin{bmatrix} v_k \\ v_j \end{bmatrix} = \begin{bmatrix} y_s & -y_m \\ -y_m & y_s \end{bmatrix} \begin{bmatrix} v_k \\ v_j \end{bmatrix}$$

$$y_s = y + \frac{1}{z} = y_c \left[\coth(\gamma\, l) - \operatorname{csch}(\gamma\, l)\right] + y_c \operatorname{csch}(\gamma\, l)$$

$$y_s = y_c \coth(\gamma\, l) \qquad\qquad y_m = y_c \operatorname{csch}(\gamma\, l)$$

$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line

$$y_s = y_c \coth(\gamma\, l) \;\Rightarrow\; \frac{dy_s}{ds} = \frac{dy_c}{ds}\coth(\gamma\, l) - y_c\,\frac{d\gamma}{ds}\, l\,\mathrm{csch}(\gamma\, l)$$

$$y_m = y_c\,\mathrm{csch}(\gamma\, l) \;\Rightarrow\; \frac{dy_m}{ds} = \frac{dy_c}{ds}\mathrm{csch}(\gamma\, l) - y_c\,\frac{d\gamma}{ds}\, l\,\mathrm{csch}(\gamma\, l)\coth(\gamma\, l)$$

$$y_c = \sqrt{\frac{Y(s)}{Z(s)}} \;\Rightarrow\; \frac{dy_c}{ds} = \frac{1}{2\gamma}\left[\frac{dY}{ds} - {y_c}^2\,\frac{dZ}{ds}\right]$$

$$\gamma = \sqrt{Z(s)\,Y(s)} \;\Rightarrow\; \frac{d\gamma}{ds} = \frac{1}{2\gamma}\left[Z\,\frac{dY}{ds} + Y\,\frac{dZ}{ds}\right]$$

$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line

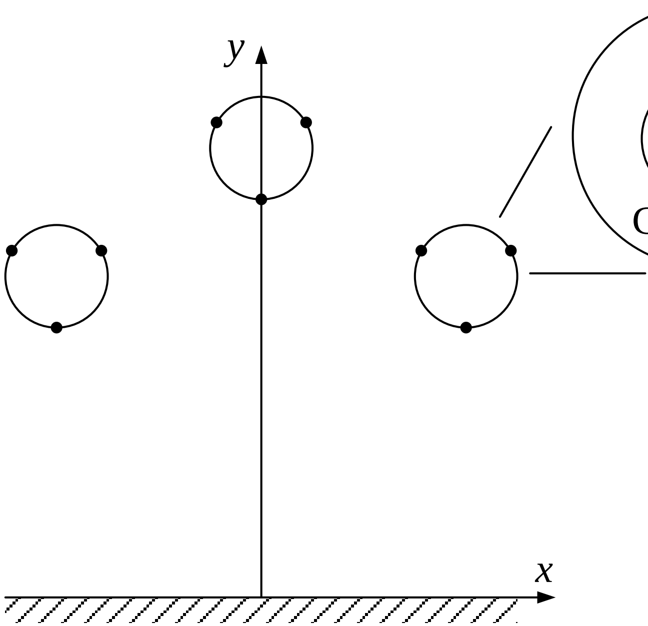
$$Y = s\,C_1 \Rightarrow \frac{dY}{ds} = C_1$$

$$Z = Z^{(e)} + Z^{(i)} + Z^{(g)} \Rightarrow \frac{dZ}{ds} = \frac{dZ^{(e)}}{ds} + \frac{dZ^{(i)}}{ds} + \frac{dZ^{(g)}}{ds}$$

$$Z^{(e)} = s\,L_1^{(e)} \Rightarrow \frac{dZ^{(e)}}{ds} = L_1^{(e)}$$

$C_1, L_1^{(e)} \rightarrow$ Positive sequence capacitance & indutance, computed by matrix reduction considering ideal conductor and soil.

$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line



$$Z^{(i)} = \frac{k(s)}{n_s} \frac{n(s)}{m(s)}$$

$$k(s) = \sqrt{\frac{s\,\mu}{\sigma}} \frac{1}{2\,\pi\,r_e}$$

$$n(s) = I_0(\rho_1)\,K_1(\rho_0) + I_1(\rho_0)\,K_0(\rho_1)$$

$$m(s) = I_1(\rho_1)\,K_1(\rho_0) + I_1(\rho_0)\,K_1(\rho_1)$$

$$\rho_0 = r_i\,\sqrt{s\,\mu\,\sigma} \qquad \rho_1 = r_e\,\sqrt{s\,\mu\,\sigma}$$

$I_0, I_1 \rightarrow$ Modified Bessel functions of first kind for integer orders 0 & 1, respectively.

$K_0, K_1 \rightarrow$ Modified Bessel functions of first kind for integer orders 0 & 1, respectively.

$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line

$$Z^{(i)} = \frac{k(s)}{n_s}\,\frac{n(s)}{m(s)} \quad \Rightarrow \quad \frac{dZ^{(i)}}{ds} = \frac{1}{n_s}\left[\frac{dk(s)}{ds}\,\frac{n(s)}{m(s)} + k(s)\,\frac{m(s)\dfrac{dn(s)}{ds} - n(s)\dfrac{dm(s)}{ds}}{m(s)^2}\right]$$

$$\frac{dk(s)}{ds} = \frac{k(s)}{2\,s}$$

$$\frac{dn(s)}{ds} = \frac{dI_0(\rho_1)}{d\rho_1}\,\frac{d\rho_1}{ds}\,K_1(\rho_0) + I_0(\rho_1)\,\frac{dK_1(\rho_0)}{d\rho_0}\,\frac{d\rho_0}{ds} + \frac{dI_1(\rho_0)}{d\rho_0}\,\frac{d\rho_0}{ds}\,K_0(\rho_1) + I_1(\rho_0)\,\frac{dK_0(\rho_1)}{d\rho_1}\,\frac{d\rho_1}{ds}$$

$$\frac{dm(s)}{ds} = \frac{dI_1(\rho_1)}{d\,\rho_1}\,\frac{d\rho_1}{d\,s}\,K_1(\rho_0) + I_1(\rho_1)\,\frac{dK_1(\rho_0)}{d\rho_0}\,\frac{d\rho_0}{d\,s} - \frac{dI_1(\rho_0)}{d\rho_0}\,\frac{d\rho_0}{ds}\,K_1(\rho_1) - I_1(\rho_0)\,\frac{dK_1(\rho_1)}{d\rho_1}\,\frac{d\rho_1}{d\,s}$$

$$\frac{d\rho_0}{ds} = \frac{\rho_0}{2\,s}$$

$$\frac{d\rho_1}{ds} = \frac{\rho_1}{2\,s}$$

$$\frac{dI_0(\rho)}{d\rho} = I_1(\rho)$$

$$\frac{dI_1(\rho)}{d\rho} = \frac{I_0(\rho) + I_2(\rho)}{2}$$

$$\frac{dK_1(\rho)}{d\rho} = -\frac{K_0(\rho) + K_2(\rho)}{2}$$

$$\frac{dK_0(\rho)}{d\rho} = -K_1(\rho)$$

$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line

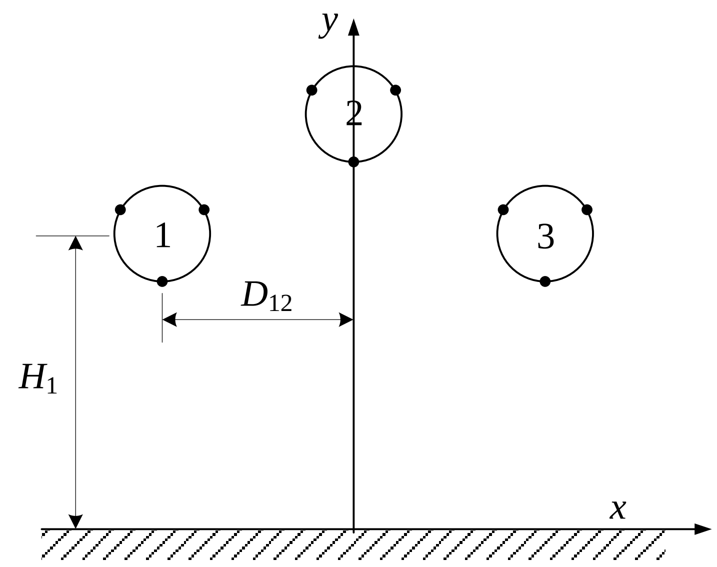
$$Z^{(g)} = s\,\frac{\mu_0}{6\,\pi}\left[\sum_{i=1}^{3}\ln\left(\frac{H_i+p}{H_i}\right)-\sum_{i=1}^{3}\sum_{\substack{j=1\\ j\neq i}}^{3}\ln\left(\frac{\sqrt{\left(H_i+H_j+2p\right)^2+D_{ij}^2}}{\sqrt{\left(H_i+H_j\right)^2+D_{ij}^2}}\right)\right] \Rightarrow$$

$$\frac{dZ^{(g)}}{ds}=\frac{\mu_0}{6\,\pi}\left[\sum_{i=1}^{3}\ln\left(\frac{H_i+p}{H_i}\right)+\frac{s}{H_i+p}\,\frac{dp}{ds}-\sum_{i=1}^{3}\sum_{\substack{j=1\\ j\neq i}}^{3}\ln\left(\frac{\hat{H}_{ij}}{\sqrt{\left(H_i+H_j\right)^2+D_{ij}^2}}\right)+\frac{s}{\hat{H}_{ij}(p)}\,\frac{d\hat{H}_{ij}(p)}{ds}\right]$$

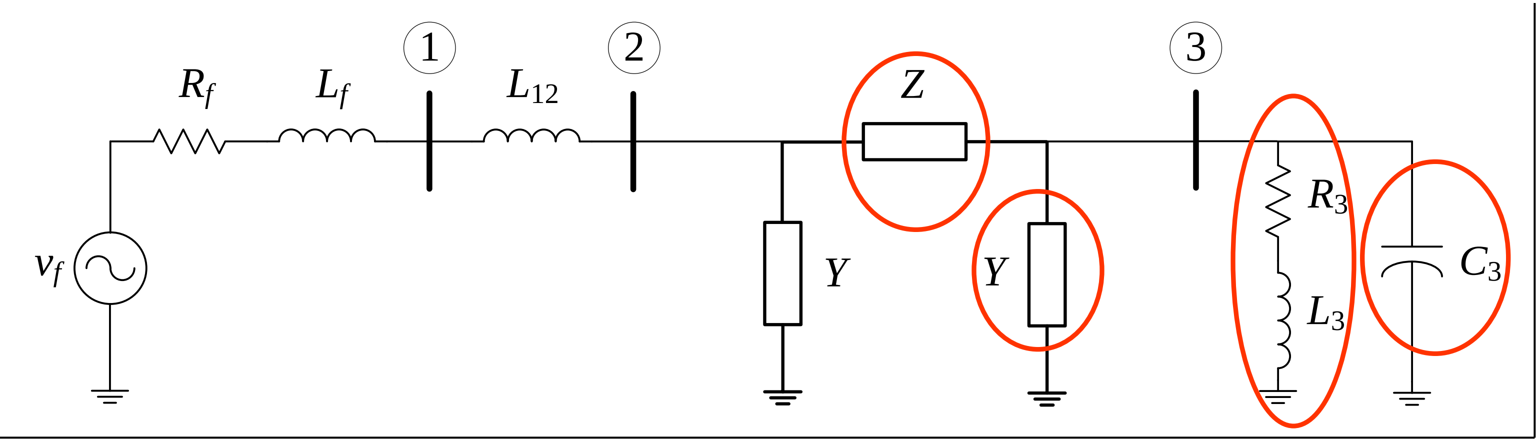
$$p=\frac{1}{\sqrt{s\,\mu\,(\sigma+s\,\varepsilon)}} \Rightarrow \frac{dp}{ds}=-\frac{\mu\,(\sigma+2\,s\,\varepsilon)\,p^3}{2}$$

$$\hat{H}_{ij}=\sqrt{\left(H_i+H_j+2p\right)^2+D_{ij}^2} \Rightarrow$$

$$\frac{d\hat{H}_{ij}(p)}{ds}=\frac{2\left(H_i+H_j+2p\right)}{\sqrt{\left(H_i+H_j+2p\right)^2+D_{ij}^2}}\,\frac{dp}{ds}$$



$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line



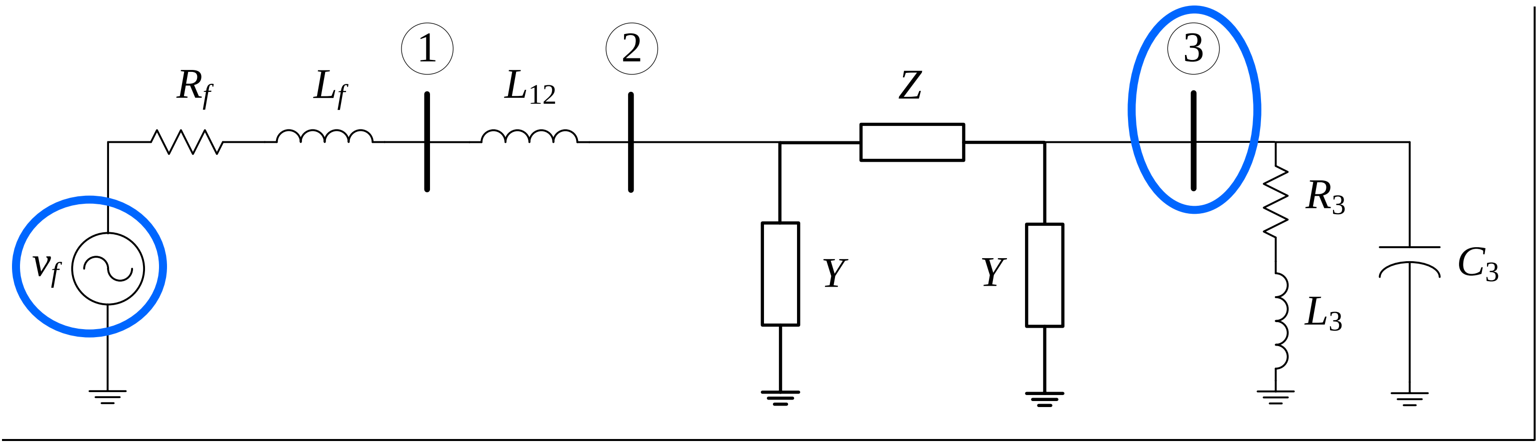
$$\mathbf{Y}(s) = \left[\begin{array}{ccc|c} y_{11} & y_{12} & 0 & -1 \\ y_{21} & y_{22} & y_{23} & 0 \\ 0 & y_{32} & y_{33} & 0 \\ \hline 1 & 0 & 0 & z_f \end{array}\right]$$

$$y_{33}(s) = \frac{1}{R_3 + s\,L_3} + s\,C_3 + \underbrace{Y + \frac{1}{Z}}_{y_c(s)\coth[\gamma(s)\,l]}$$

$$\frac{d\mathbf{Y}(s)}{ds} = \left[\begin{array}{ccc|c} dy_{11}/ds & dy_{12}/ds & 0 & 0 \\ dy_{21}/ds & dy_{22}/ds & dy_{23}/ds & 0 \\ 0 & dy_{32}/ds & dy_{33}/ds & 0 \\ \hline 0 & 0 & 0 & dz_f/ds \end{array}\right]$$

$$\frac{dy_{33}}{ds} = \frac{-L_3}{(R_3 + s\,L_{12})^2} + C_3 + \frac{d}{ds}\{\, y_c(s)\coth[\gamma(s)\,l]\,\}$$

$\mathbf{Y}(s)$ Matrix – Distrib. Param. Transmission Line



$$\left[\begin{array}{c:c:c|c} y_{11} & y_{12} & 0 & -1 \\ \hdashline y_{21} & y_{22} & y_{23} & 0 \\ \hdashline 0 & y_{32} & y_{33} & 0 \\ \hline 1 & 0 & 0 & z_f \end{array}\right]\left[\begin{array}{c} v_1 \\ \hdashline v_2 \\ \hdashline v_3 \\ \hline i_f \end{array}\right] = \left[\begin{array}{c} 0 \\ \hdashline 0 \\ \hdashline 0 \\ \hdashline 1 \end{array}\right] v_f$$

$$v_3 = \left[0 \;\vdots\; 0 \;\vdots\; 1 \;|\; 0\right]\left[\begin{array}{c} v_1 \\ \hdashline v_2 \\ \hdashline v_3 \\ \hline i_f \end{array}\right]$$

$$\mathbf{Y}(s)\,\mathbf{x} = \mathbf{b}\, v_f$$

$$v_3 = \mathbf{c}^t\,\mathbf{x}$$

$$v_3 = \mathbf{c}^t\,\mathbf{Y}(s)^{-1}\,\mathbf{b}\, v_f$$

$$G_{31}(s) = \frac{v_3}{v_1\left(= v_f\right)} = \mathbf{c}^t\,\mathbf{Y}(s)^{-1}\,\mathbf{b}$$

$$\mathbf{b}^t = \left[0 \;\vdots\; 0 \;\vdots\; 0 \;|\; 1\right]$$

$$\mathbf{c}^t = \left[0 \;\vdots\; 0 \;\vdots\; 1 \;|\; 0\right]$$

Sequential MIMO Dominant Pole Algorithm (SMDPA)

SMDPA – Fundamental Concepts (1/3)

- Direct Matrix

$$\mathbf{H}(s) = \underbrace{\mathbf{C}^T \mathbf{Y}(s)^{-1} \mathbf{B}}_{\mathbf{D}_i} + \mathbf{D}_e$$

$$\mathbf{D} = \mathbf{D}_i + \mathbf{D}_e$$

- Partial Fraction Expansion

$$\mathbf{H}(s) = \sum_{i=1}^{\infty} \frac{\mathbf{R}_i}{s - \lambda_i} + s\mathbf{K} + \mathbf{D}_i + \mathbf{D}_e$$

$$\mathbf{K} = \lim_{s \to \infty} \frac{d\mathbf{H}(s)}{ds}$$

$$\mathbf{D}_i = \lim_{s \to \infty} \mathbf{H}(s) - s\mathbf{K} - \mathbf{D}_e$$

SMDPA – Fundamental Concepts (2/3)

- Strictly Proper part of $\mathbf{H}(s)$

$$\mathbf{H}(s) = \underbrace{\sum_{i=1}^{\infty} \frac{\mathbf{R}_i}{s-\lambda_i}}_{\hat{\mathbf{H}}(s)=\mathbf{H}(s)-s\mathbf{K}-\mathbf{D}} + s\mathbf{K} + \mathbf{D}$$

- Dominant Pole

Let $\lambda_k = \alpha_k + j\,\beta_k$ be a pole with an associated residue matrix $\mathbf{R}_k$, then:

$$\hat{\mathbf{H}}(j\beta_k) = -\frac{\mathbf{R}_k}{\alpha_k} + \sum_{\substack{i=1\\ i\neq k}}^{\infty} \frac{\mathbf{R}_i}{j\beta_k - \lambda_i}$$

The pole λ_k will be dominant in $\mathbf{H}$(s) if the magnitude of ($\|\mathbf{R}_k\|_2$ / $|\alpha_k|$) is sufficiently large so as to cause a peak in the plot of $\sigma_{max}[\hat{\mathbf{H}}(j\omega)]$ in the close neighborhood of the frequency β_k.

SMDPA – Fundamental Concepts (3/3)

- Reduced Order Model (ROM)

$$\mathbf{H}(s) \cong \mathbf{H}_N(s) = \sum_{\substack{i=1 \\ \lambda_i \in \Omega}}^{N} \frac{\mathbf{R}_i}{s-\lambda_i} + s\,\mathbf{K} + \mathbf{D}$$

$\Omega \rightarrow$ Set of N dominant poles and associated residue matrices.

- MIMO ROM Deviation TF

$$\overline{\mathbf{H}}(s) = \mathbf{H}(s) - \mathbf{H}_N(s) = \mathbf{H}(s) - s\,\mathbf{K} - \mathbf{D} - \sum_{\substack{i=1 \\ \lambda_i \in \Omega}}^{N} \frac{\mathbf{R}_i}{s-\lambda_i}$$

- Norm of MIMO ROM Deviation TF

$$\varepsilon_{MOR}(j\omega) = \sigma_{\max}\left[\overline{\mathbf{H}}(j\omega)\right]$$

SMDPA – Newton Method (1/3)

- The set of dominant poles of $\mathbf{H}(s)$ may be efficiently computed only when eliminating from $\mathbf{H}(s)$:
  - The N previously computed poles (deflation);
  - Matrices $\mathbf{K}$ and $\mathbf{D}$.
- The Newton method should therefore be applied to the MIMO ROM deviation TF:

$$\overline{\mathbf{H}}(s) = \mathbf{H}(s) - s\,\mathbf{K} - \mathbf{D} - \sum_{\substack{i=1 \\ \lambda_i \in \Omega}}^{N} \frac{\mathbf{R}_i}{s - \lambda_i}$$

SMDPA – Newton Method (2/3)

- Pole of $\overline{\mathbf{H}}(s)$

$$\lim_{s\to\lambda} \mu_{\min}\left[\overline{\mathbf{H}}(s)^{-1}\right] = 0$$

$\mu_{\text{mim}} \to$ minimum eigenvalue of $\overline{\mathbf{H}}(s)^{-1}$

$\mathbf{v}_{\min}, \mathbf{w}_{\min} \to$ eigenvectors associated with $\mu_{\min}$.

- Newton equationing

$$f(s) = \mu_{\min}(s) = 0$$

$$s^{(k+1)} = s^{(k)} + \Delta s$$

$$\Delta s^{(k)} = \frac{1}{\mu_{\min}\left(s^{(k)}\right) \mathbf{w}_{\min}\left(s^{(k)}\right)^{*} \dfrac{d\overline{\mathbf{H}}(s^{(k)})}{ds} \mathbf{v}_{\min}\left(s^{(k)}\right)}$$

($\mu_{\text{mim},}$ $\mathbf{v}_{\text{min},}$ $\mathbf{w}_{\min}$) function **eig** of Matlab.

$$\frac{d\overline{\mathbf{H}}(s)}{ds} = \frac{d\mathbf{H}(s)}{ds} - \frac{d\mathbf{H}_N(s)}{ds}$$

$$\frac{d\mathbf{H}(s)}{ds} = -\mathbf{X}_{\mathbf{C}}^{T}(s)\frac{d\mathbf{Y}(s)}{ds}\mathbf{X}_{\mathbf{B}}(s)$$

$$\frac{d\mathbf{H}_N(s)}{ds} = -\sum_{\substack{i=1 \\ \lambda_i \in \Omega}}^{N} \frac{\mathbf{R}_i}{(s-\lambda_i)^2} + \mathbf{K}$$

$$\mathbf{Y}(s)\mathbf{X}_{\mathbf{B}}(s) = \mathbf{B}$$

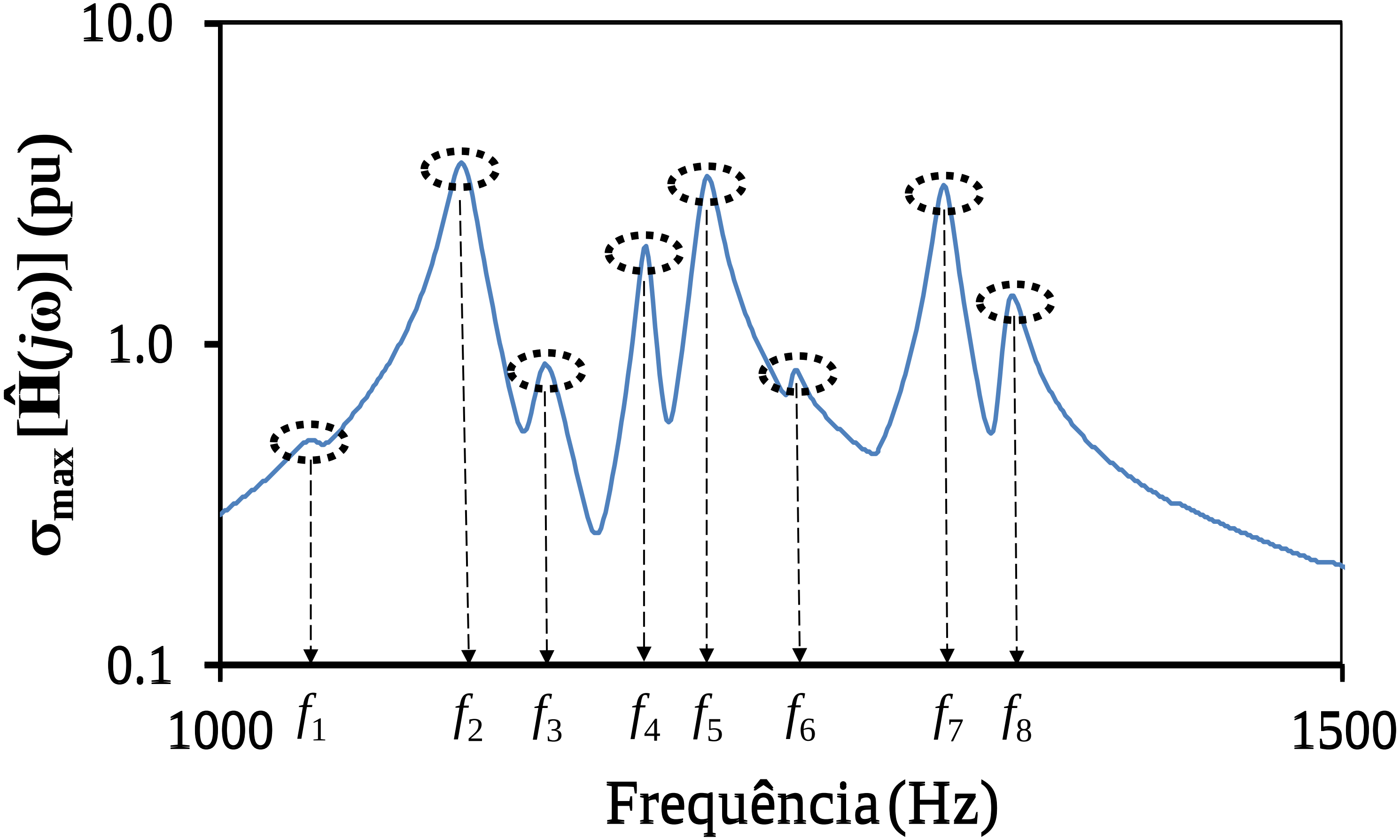
$$\mathbf{Y}(s)^{T}\mathbf{X}_{\mathbf{C}}(s) = \mathbf{C}$$

The sparse solution of these two matrix equations sistemas require a single LU factorization and various solves.

SMDPA – Newton Method (3/3)

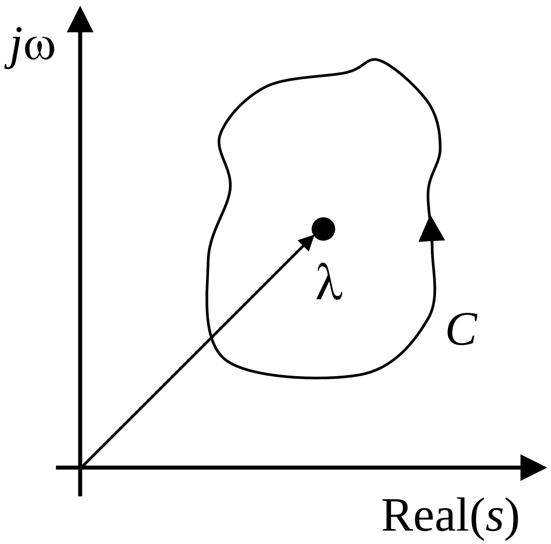
- Determining the initial pole estimates

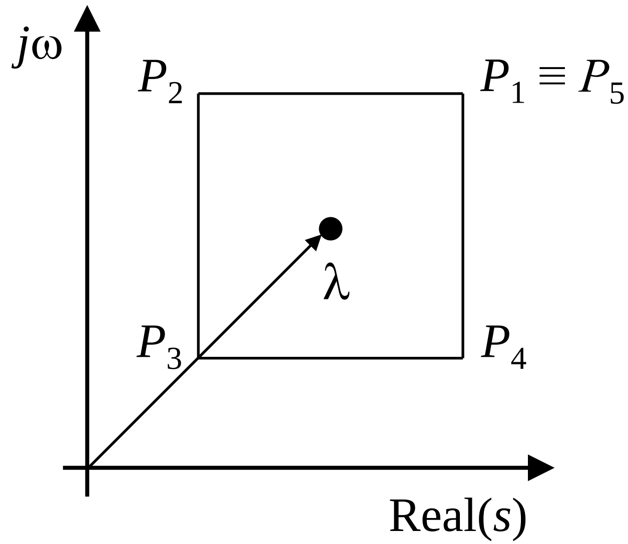
$$\mathbf{s}^{(0)} = 2\pi j \begin{bmatrix} f_1 & \cdots & f_8 \end{bmatrix}$$



SMDPA – Pole Residue Matrix (1/2)

- Computation of the Residue Matrix for a Pole
  - Definition of the Integration Curve



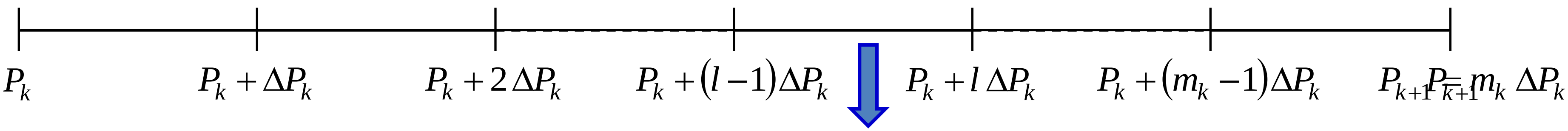


$$\mathbf{R} = \frac{1}{2\pi j} \oint_C \mathbf{H}(s)\, ds$$

$$\oint_C \mathbf{H}(s)\, ds = \sum_{k=1}^{4} \int_{P_k}^{P_{(k+1)}} \mathbf{H}(s)\, ds$$

SMDPA – Pole Residue Matrix (2/2)

- Legendre-Gauss Method with Error Control



$$s = P_k + \left[\, l + 0.5\,(\xi - 1)\,\right] \Delta P_k$$

$$\oint_C \mathbf{H}(s)\,ds = \sum_{k=1}^{4} \sum_{l=1}^{m_k} \int_{P_k+(l-1)\,\Delta P_k}^{P_k + l\,\Delta P_k} \mathbf{H}(s)\,ds \cong \sum_{k=1}^{4} \frac{\Delta P_k}{2} \sum_{l=1}^{m_k} \sum_{i=1}^{M} w_i\ \mathbf{J}(\xi_i, l, P_k)$$

$$\mathbf{J}(\xi, l, P_k) = \mathbf{H}\left(P_k + \left[\, l + 0.5\,(\xi - 1)\,\right] \Delta P_k \right)$$

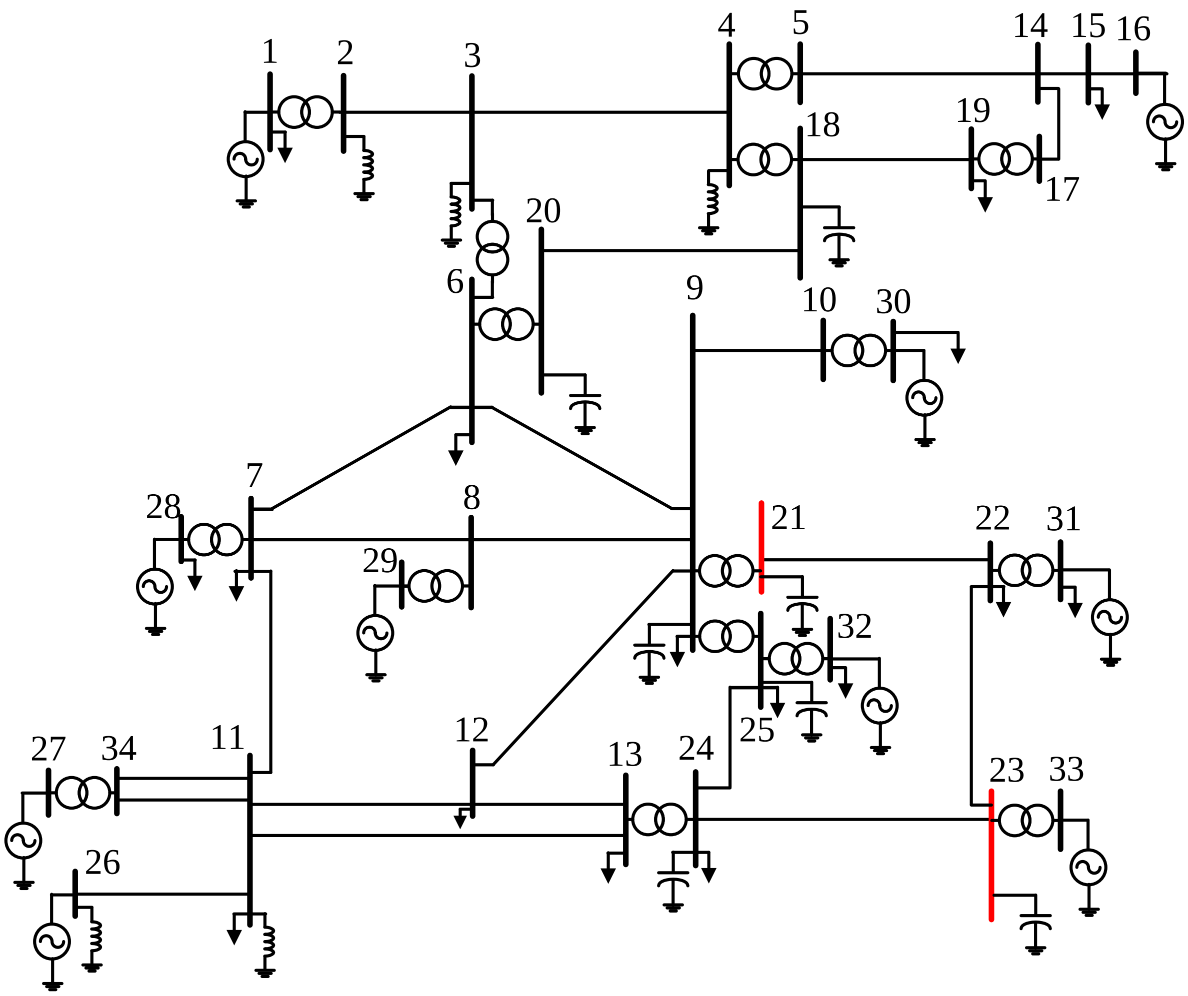
$$\mathbf{R} \cong \frac{1}{2\pi j} \sum_{k=1}^{4} \frac{\Delta P_k}{2} \sum_{l=1}^{m_k} \sum_{i=1}^{M} w_i\ \mathbf{J}(\xi_i, l, P_k) \Rightarrow \mathbf{R}_k \cong \frac{1}{2\pi j} \frac{\Delta P_k}{2} \sum_{l=1}^{m_k} \sum_{i=1}^{M} w_i\ \mathbf{J}(\xi_i, l, P_k),\ k = 1, \ldots, 4$$

- Determining the Error

$$m_k = 2^q,\ \ q = 0, 1, 2, \ldots \Rightarrow \varepsilon_k = \left\| \mathbf{R}_k^{(q+1)} - \mathbf{R}_k^{(q)} \right\|_2 \Big/ \left\| \mathbf{R}_k^{(q+1)} \right\|_2 \Rightarrow \varepsilon = \sum_{k=1}^{4} \varepsilon_k \le 4\,\varepsilon_{\max}$$

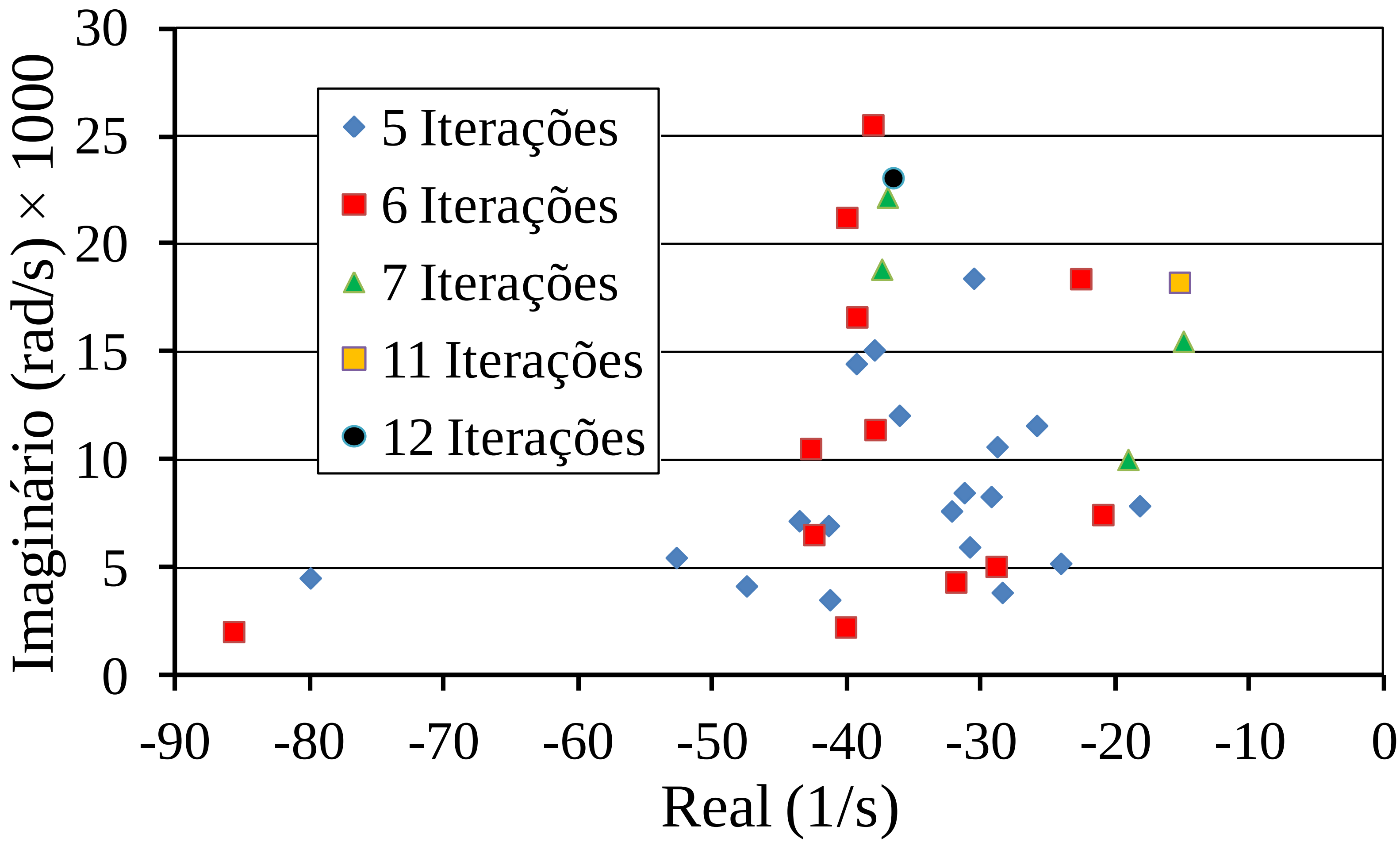
SMDPA – 34-bus Test System Results (1/4)

- The 34-bus test system with 25 distributed parameter TLs



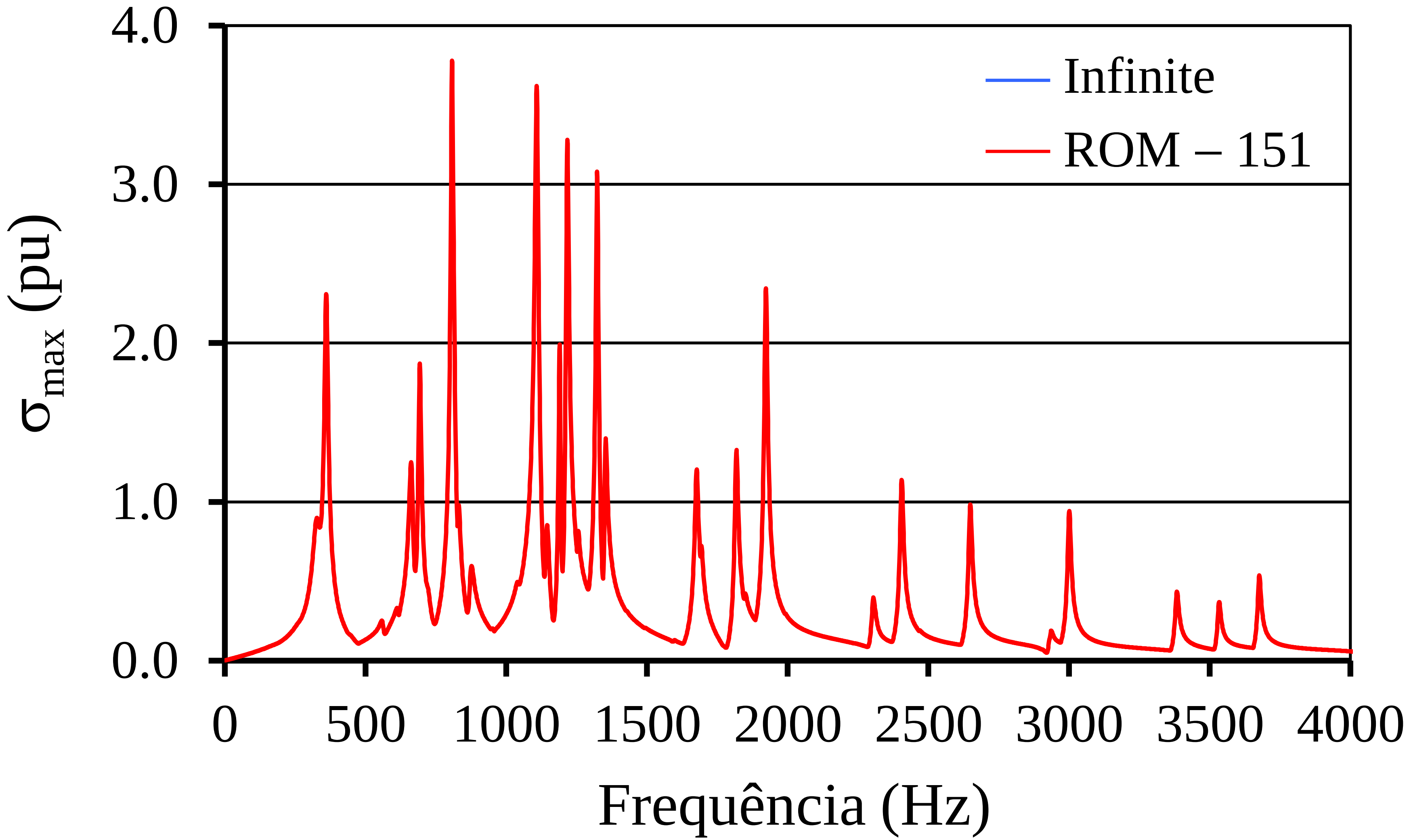
Element	Quant.
Buses	34
TLs	25
Branches	12
Trafos	16
Loads	16
Generators	10

SMDPA – 34-bus Test System Results (2/4)

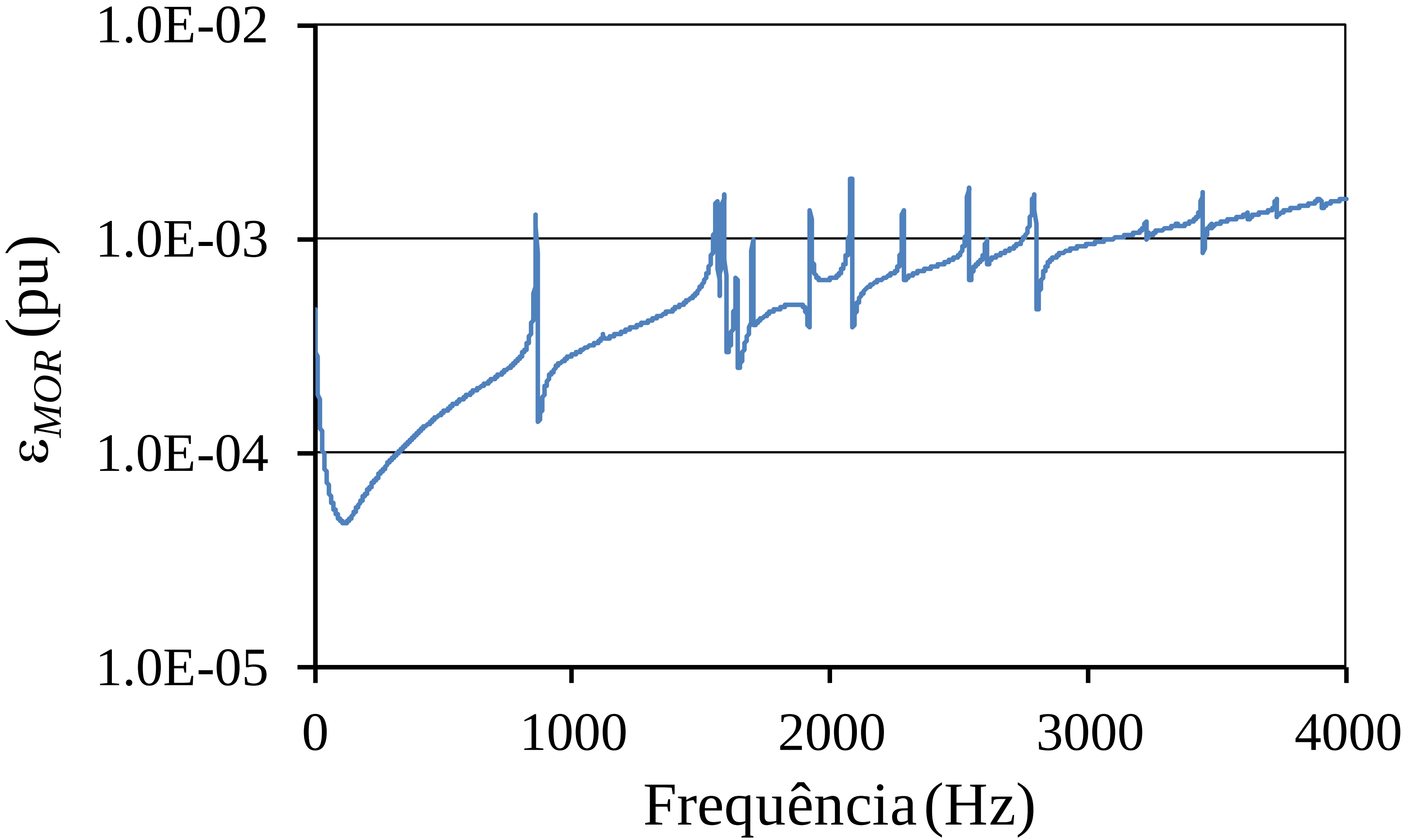


Most dominant pole spectrum for 34-bus system (2x2) TF. Color code identifies iterations required for SMDPA convergence from the initial set of 37 estimates

SMDPA – 34-bus Test System Results (3/4)



SMDPA – 34-bus Test System Results (4/4)



SMDPA - Infinite × Finite Systems (1/5)

- ROM and LMA Errors

  - ROM

$$\overline{\mathbf{H}}(s) = \underbrace{\mathbf{H}(s)}_{\mathbf{C}^T \mathbf{Y}(s)^{-1} \mathbf{B} + \mathbf{D}_e} - \underbrace{\mathbf{H}_N(s)}_{\sum_{\substack{i=1 \\ \lambda_i \in \Omega}}^{N} \frac{\mathbf{R}_i}{s - \lambda_i} + s\mathbf{K} + \mathbf{D}}$$

  - LMA – Linear Model (finite) Aproximation

$$\overline{\mathbf{H}}_L(s) = \mathbf{H}(s) - \underbrace{\mathbf{H}_L(s)}_{\mathbf{C}_L^T (s\mathbf{T} - \mathbf{A})^{-1} \mathbf{B}_L + \mathbf{D}_e}$$

  - Error Measures for ROM and LMA

$$\bar{\varepsilon}_{MOR} = \frac{\int_0^{\omega_f} \sigma_{\max}\left[\overline{\mathbf{H}}(j\omega)\right] d\omega}{\int_0^{\omega_f} \sigma_{\max}\left[\mathbf{H}(j\omega)\right] d\omega} \times 100\,\%$$

$$\bar{\varepsilon}_{MLA} = \frac{\int_0^{\omega_f} \sigma_{\max}\left[\overline{\mathbf{H}}_L(j\omega)\right] d\omega}{\int_0^{\omega_f} \sigma_{\max}\left[\mathbf{H}(j\omega)\right] d\omega} \times 100\,\%$$

SMDPA - Infinite × Finite Systems (2/5)

- How many π circuits to use in the TL models of the 34-bus system?
- The tables below compare the performances of finer LMAs × ROM-151

Statistics for LMA models

n_π	n_D	n_L	n_L / n
300	15063	15182	345.05
400	20063	20182	458.68
500	25063	25182	572.32
600	30063	30182	685.95

Error Measures for LMA and ROM

n_π	$\bar{\varepsilon}_{MLA}(\%)$	$\bar{\varepsilon}_{MOR}(\%)$
300	4.59×10^{-1}	
400	2.58×10^{-1}	1.94×10^{-1}
500	1.65×10^{-1}	
600	1.15×10^{-1}	

n_π $\rightarrow$ Number of π circuits per TL

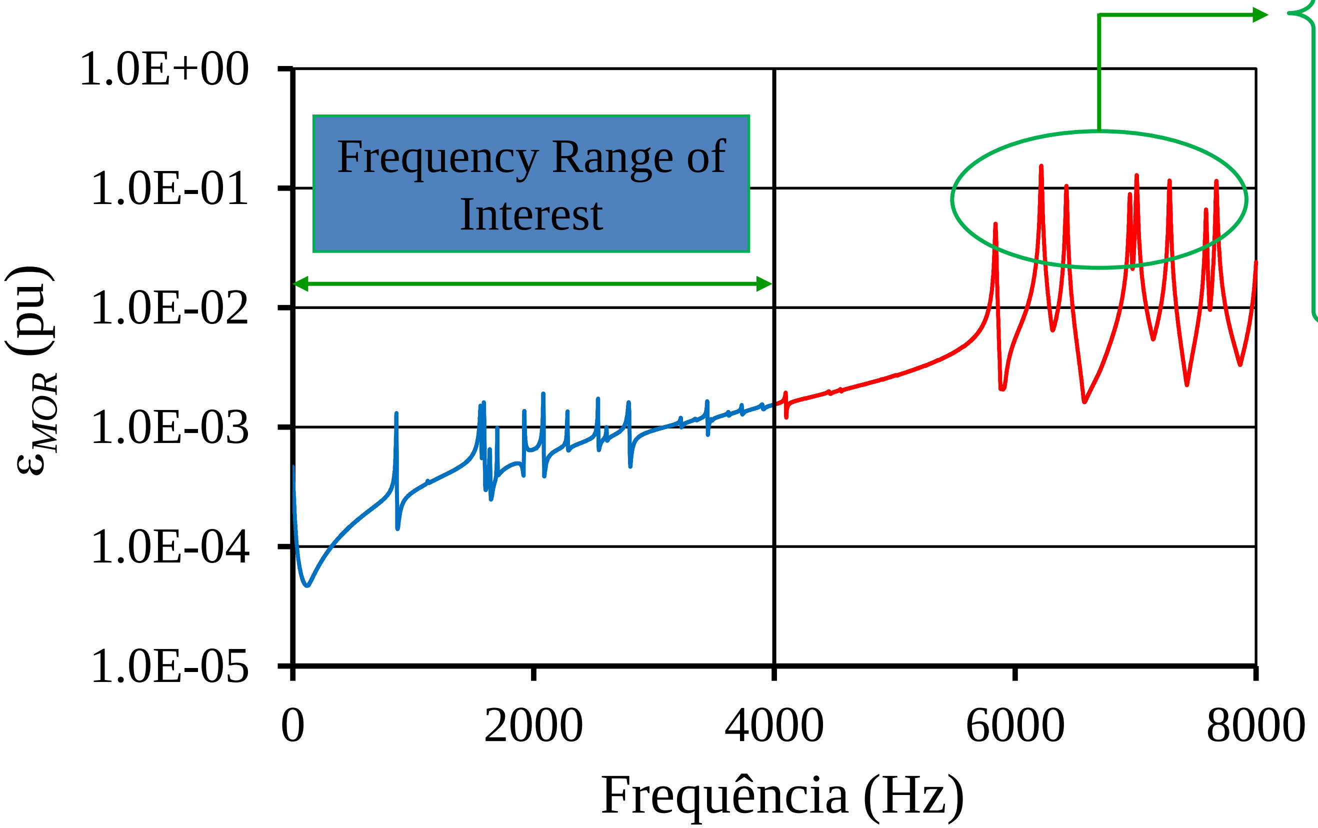
n_D $\rightarrow$ Number of differencial equations

n_L $\rightarrow$ Dimension of matrices **A** and **T**

n $\rightarrow$ Dimension of matrix $\mathbf{Y}(s)$ (= nº of buses + nº of voltage sources = 44)

SMDPA - Infinite × Finite Systems (3/5)

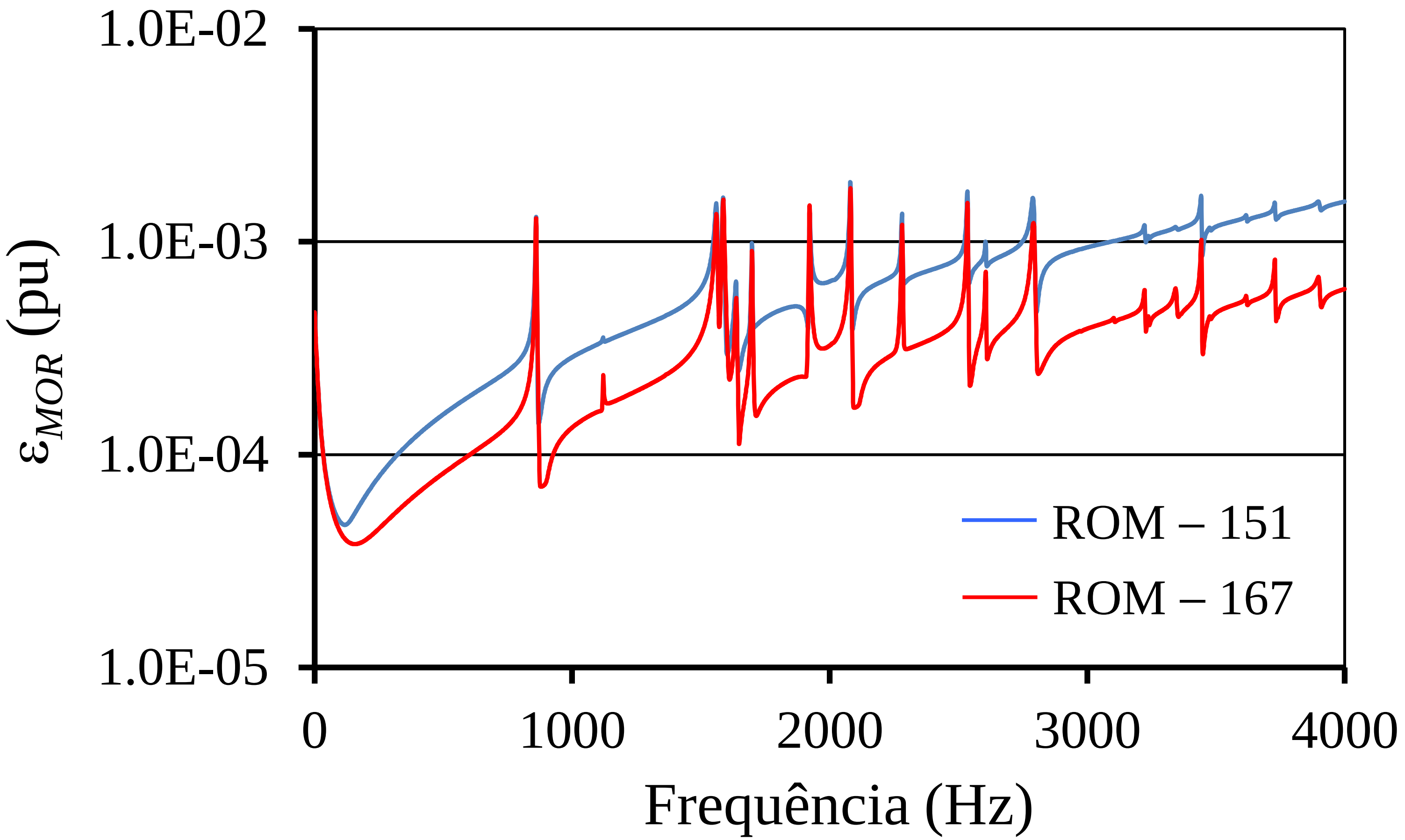
- Simple procedure for improving ROM Fidelity of infinite systems
  - ROM – 151 for MIMO TF of 34-bus system



These eight spikes are due to the 8 pairs of poles that are dominant in the adjacent 4 - 8 KHz region, and cause the error curve within the 0 – 4 KHz window to rise.

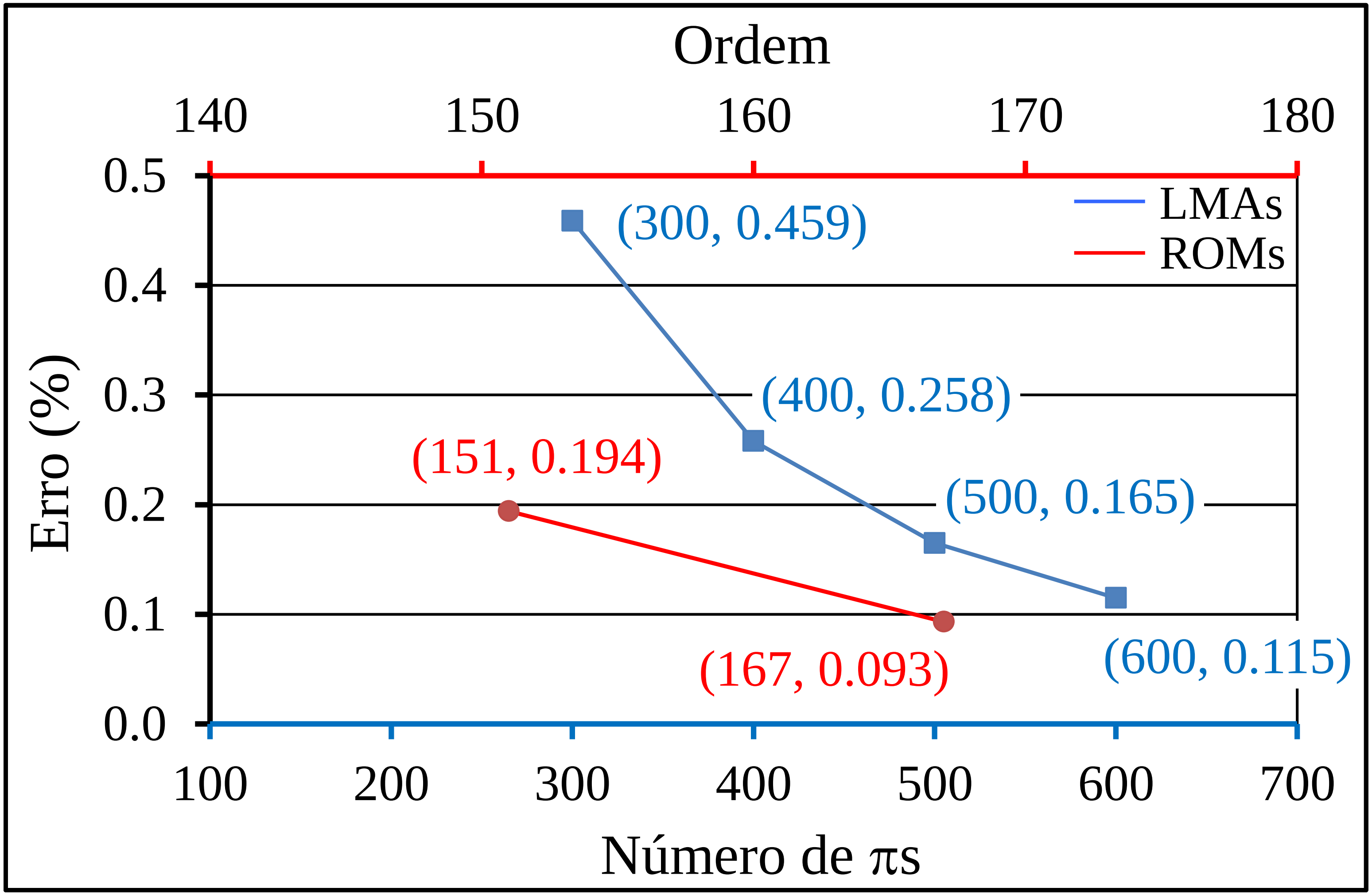
SMDPA - Infinite × Finite Systems (4/5)

- Assessing ROM Fidelity for the MIMO TF of 34-bus System
  - ROM - 151 × ROM - 167



SMDPA - Infinite × Finite Systems (5/5)

- Comparing the Performances of LMAs × ROMs



Conclusions for Part II (1/2)

- Electrical network modeling with its RLC series and paralell components, current and voltage sources, in the DS and Y(s) matrix formulations;
- Development of the first reliable Newton algorithm for computing the dominant poles of SISO and MIMO TFs of infinite systems (SMDPA). The method's reliability comes from the very effective pole deflation procedure and the accurate computation of the pole residue matrices;
  - The residue is numerically computed as the path integral around the pole which was here obtained by the Legendre-Gauss quadrature method.
- The modeling accuracy of the DS and Y(s) formulations was verified by the close matching between their simulation results and those obtained with ATP or PSCAD for various test systems.
- Y(s) allows the exact modeling of linear systems incorporating time delay.

Conclusions for Part II (2/2)

- SMDPA yields high fidelity ROMs over a specified frequency window, for use as equivalents in transmission network electromagnetic transient studies.
- Multi-bus ROMs produced by SMDPA directly from infinite system models are a more practical option than LMA (Linear Matrix Approximation) models;
- In attempting to obtain accuracy over a wider frequency range, LMA models may soon reach uncomfortably large dimensions and present severe numerical stiffness.
- Application of SMDPA to other areas of engineering, physics and mathematics is yet to be explored.

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Thank you!

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Cepel