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Social Opinion Dynamics: Agreement and Disagreement

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Outline

1. Background
2. Opinion dynamics
3. Bounded confidence model
4. Our Results
5. Conclusions

1. Background

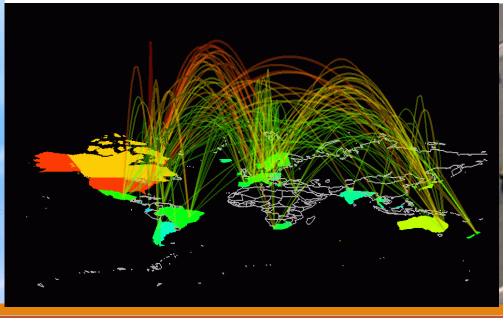
Social networks become a hot topic

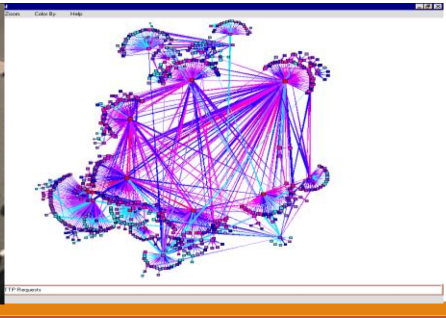
- Applications: political voting, terrorist war, mass media, e-business, public innovation, smart cities, …



Why now?

- Development of information/data technique: Big data, digital media, cloud computation, agent-based models, distributed algorithms …
  →Google, Amazon, Facebook, Baidu, …
- Interdisciplinary research: network science, math, sociology, psychology, economics, …

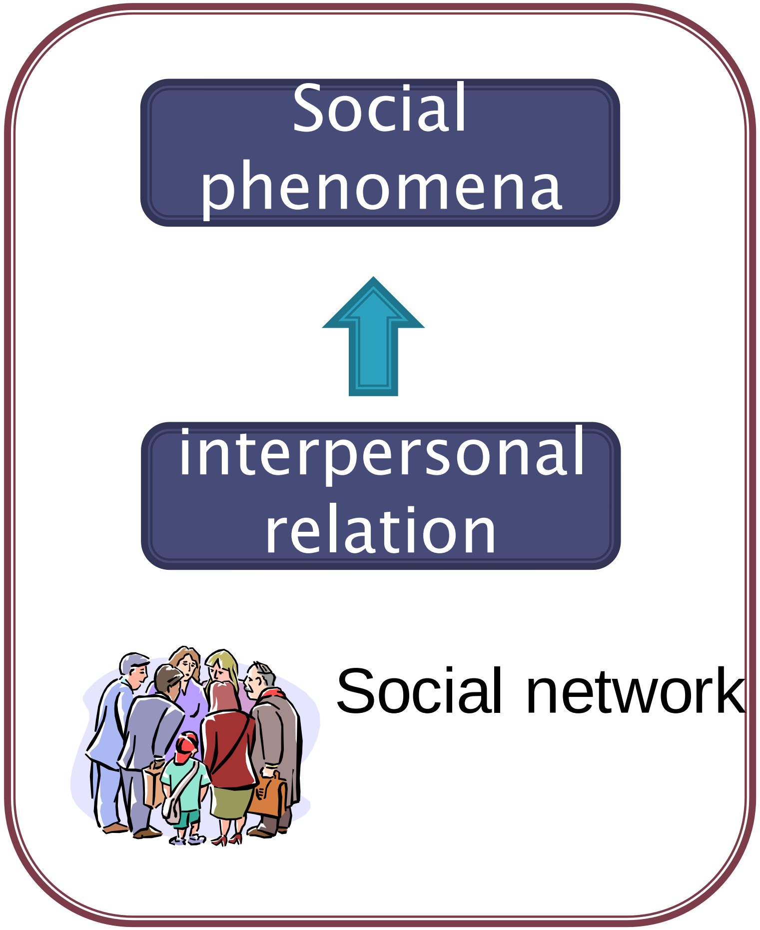


Social networks

1. Systems effect: local interaction → collective phenomena (agreement or disagreement)

2. Hierarchical structure: individual, community, …, the whole society

3. Intervention policy: various ways implemented in social networks.



Opinion dynamics

- Social opinion dynamics ← changes of opinion/belief/attitude in a group or society
- From sociological/psychological viewpoints
  - Social power (1950’s)
  - Social psychology (1960’s)
  - Crowd polarization, voting (1970’s)
  - Social structure (1980’s) …

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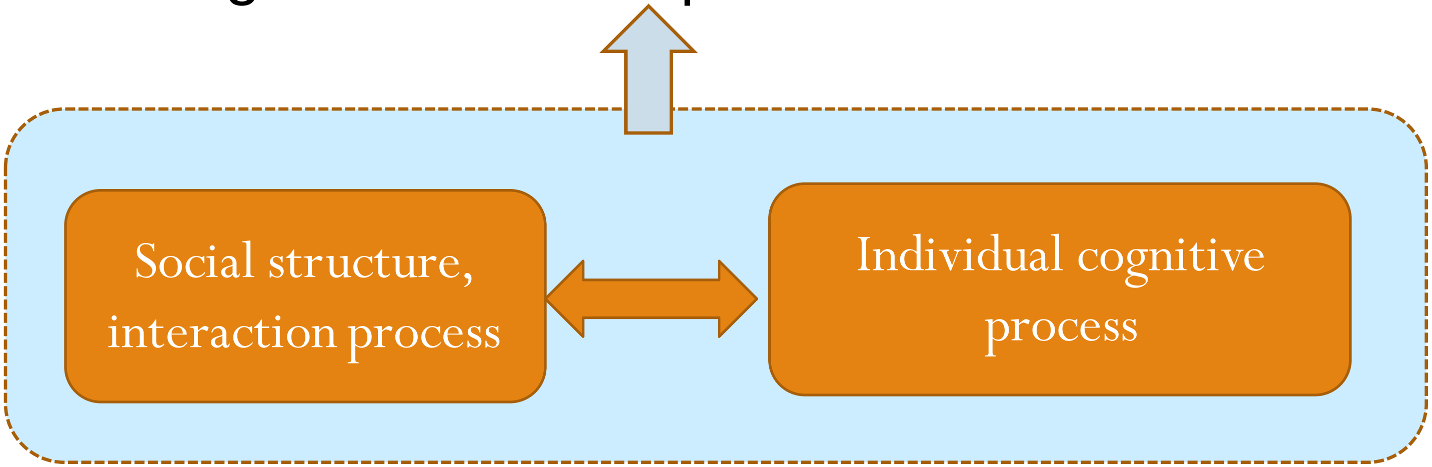
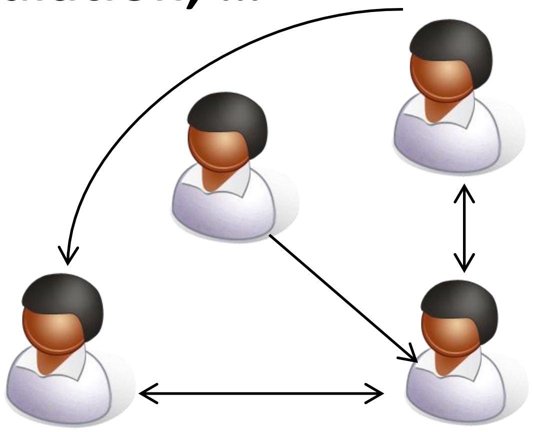
2. Opinion Dynamics (OD)

How a social group, with (initial) individual opinions, reaches a steady-state collective opinion pattern by individual cognition and interpersonal relations.



Problems of opinion dynamics

- **Opinion Propagation: How one's opinion influences others? How an individual opinion becomes public? …**
- **Opinion Evolution: How crowd polarization appears? How the opinion fluctuates in an election? …**
- **Opinion Intervention: censorship, manipulation, …**
- ……



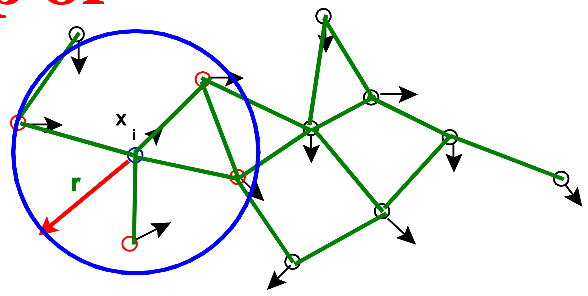
Engineerization of OD

- New Era: *"The convergence of social and technological networks"* (Jon Kleinberg)
- "Engineering" by math and data techniques for underlying opinion mechanics:
  - Measurement of opinions
  - Modeling of OD (update law, initial condition):
    - Multi-agent networks
    - Hydrodynamics: Partial differential equations

Simple models → complex phenomena

Multi-agent system (MAS)

Agent → multi-agent system: a group of subsystems



Neighbor Graph

Agent Dynamics = **a** + **b**

a: combination of neighbor information

b: private source or prejudice or free will …

→ stubborn agent (leader) if **a** =0;

→ regular agent (follower) if **b** =0

Consensus/agreement/synchronization: a basic problem → All or some variables of the agents become the same (thousands of consensus papers each year!)

Good time to study …

100 years ago, emerging of mathematical biology

- Luther: Biological travelling waves in bio-chemical reaction, 1906
- Lotka: Elements of physical biology, 1925
- Enzyme kinetics: Mechaelis-Menten enzyme reaction model, 1913
- Interacting population: Lotka-Volterra predator-prey model, 1926
- Mathematical theory for epidemics: Kermack-McKendrick SIR model, 1927
- ……

Start with simple models

How to start mathematical analysis on OD?

- **<u>French model</u>:** $P(t+1)=AP(t)$, **where** A **is the influence matrix,** P **a matrix with** p_{ij} **describing the opinion of agent *i* about agent *j*,** by French, 1956

- **<u>DeGroot model</u>:** $x(t+1)=Wx(t)$, **where** W **is the update matrix,** x **is a vector with** x_i **as the opinion value of agent *i*,** by DeGroot, 1974

- **<u>Voter model</u>:** $x_i=1$ **or** -1**, an agent updates its opinion following the neighbor it selects each time,** by Clifford & Sudbury, 1973

Good time to study …

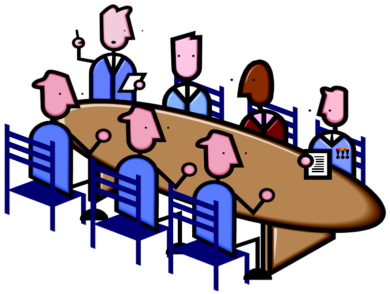
Around the beginning of this century, more and more models developed for OD (to replace old and simple models)

- Axelrod model, 1997
- Friedkin or Friedkin-Johnsen (FJ) model, 1999
- Sznajd model, 2000
- Deffuant or Deffuant-Weisbuch (DW) model, 2000
- Krause or Hegelmann-Krause (HK) model, 2002
- …… more to come

New History!

Classifications of OD

- by opinion measurement: discrete value, continuous value, vector
- by neighbor definition: based on graph or bounded confidence
- by mathematical description: deterministic or stochastic
- by interaction type: directed, undirected, or antagonistic
- by update moment: synchronous or asynchronous
- … …

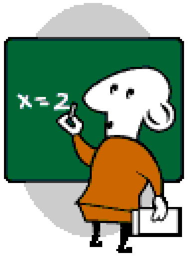


Examples

<u>DeGroot model, Friedkin model</u>: well-known deterministic continuous models

<u>Voter model</u>: a stochastic discrete model.

<u>Axelrod model</u>: a vector-valued model, to describe the opinion about multi-dimensional (entangled) issues.



Interesting cases

Opinion propagation: Complex contagion (regularity of graphs increases social affirmation)

Centola (2010): the spread of behavior in an online social network experiment, *Science*.

Opinion evolution: Reverting in the edition of Wikipedia, verified by modified DW models

Iba et al (2010) and Torok (2013) studied Wikipedia reverting behavior to match real data.

Opinion Intervention: War with Iraq in 2003: from "Unjustified" to "Justified" in a short period

Tempo, Friedkin, et al (2016): how Powel's speech led to that the preemptive attack of Iraq is a just war

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1. Background
2. Opinion Dynamics
3. **Bounded confidence model**
4. Our Results
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3. Bounded-Confidence Model

Given a bounded confidence/trust range, an agent's neighbors are agents whose opinion values are located in its confidence range → confidence/trust defined by opinion difference, not links.

Two mathematical models based on social studies

- Hegselmann-Krause (HK) or Krause model -- average
- Deffuant-Weisbuch (DW) or Deffuant model -- gossip

Basic description

- Consider n persons (agents)
- Each agent has its opinion, described by a real number x_i
- The initial opinion values are randomly distributed in a bounded interval (for example, in [0,1], where 0 and 1 represent the two extreme opinion values)
- Confidence bound/radius ε defines a neighbor set
- Average all the opinions of the neighbors (HK); count the opinion if the randomly selected agent is a neighbor (DW)

HK Model

- R. Hegselmann and U. Krause
  - Article “Opinion dynamics and bounded confidence models”, 2002
  - Book “Opinion Dynamics Driven by Various Ways of Averaging”, Kluwer Academic Publishers 2004.
- Hegselmann-Krause (HK) Model:

$$x_i(t+1) = |\mathcal{N}(i, x(t))|^{-1} \sum_{j \in \mathcal{N}(i,x(t))} x_j(t), \quad i = 1, \ldots, n$$

with the opinion value of agent i as $x_i(t) \in [0,1]$,

$$\mathcal{N}(i, x(t)) = \{1 \leq j \leq n \mid |x_j(t) - x_i(t)| \leq \epsilon\}$$

$\epsilon \in (0, 1]$ is the confidence bound/radius to define neighbors

DW Model

- G. Deffuant, et al, “Mixing beliefs among interacting agents”, 2000

  G. Weisbuch, G. Deffuant, et al, “Meet, discuss and segregte“, 2002.

- Deffuant-Weisbuch (DW) Model:

$$x_i(t+1) = x_i(t) + \gamma \mathbb{1}_{\{|x_j(t)-x_i(t)|\leq\epsilon\}}(x_j(t) - x_i(t));$$
$$x_j(t+1) = x_j(t) + \gamma \mathbb{1}_{\{|x_j(t)-x_i(t)|\leq\epsilon\}}(x_i(t) - x_j(t))$$

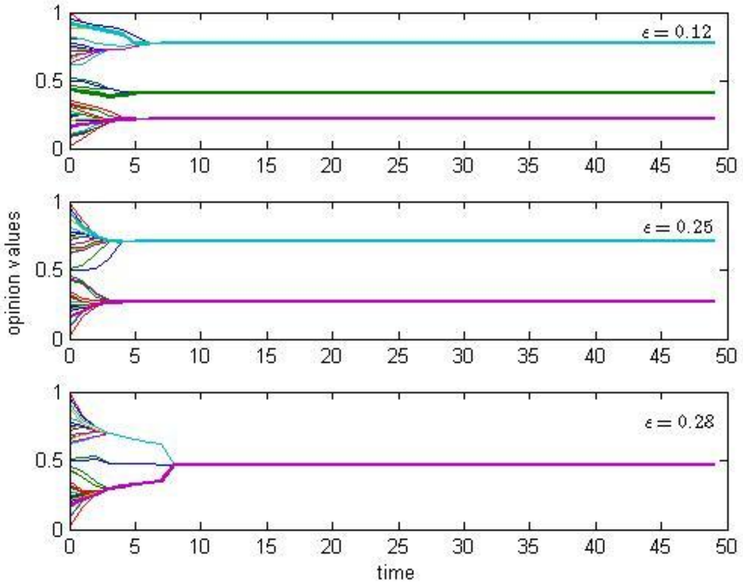
where $\mathbb{1}$ is the indicator function, i, j are randomly selected each time, and $\gamma \in (0, 1)$ is the weight.

HK vs. DW

HK model is a deterministic continuous model with confidence bound, undirected interaction, and synchronous update

Large confidence bounds → consensus/agreement;

Small bounds → fragmentation (multiple opinion subgroup)

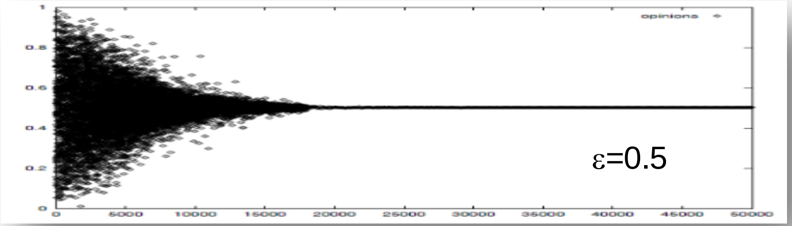


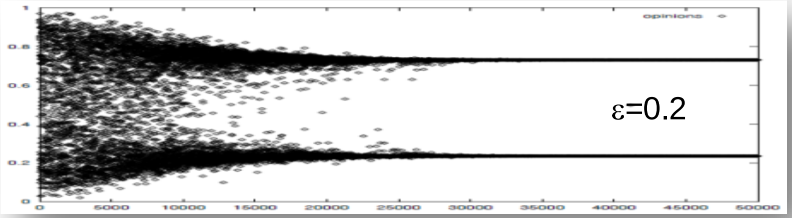
HK vs. DW

DW model is a stochastic continuous model with confidence bound, undirected interaction, and asynchronous update.

Larger bounds → agreement

Agreement is harder to be achieved and convergence is slower in the DW model





Variants of HK model

Constant confidence bound → time-varying confidence bound: vanishing bound (Girard et al, 2011)

Constant weight → changing weights in the confidence range (Motsch and Tadmor, 2014)

Homogeneous (undirected interaction) → heterogeneous (directed interaction): different agents have different confidence bounds, that is, different ε_i (Lorenz, 2007)

……

Variants of DW model

Symmetric → asymmetric: when agent *i* selects *j*, *j* may not select *i*, and therefore, the connection is directed (Zhang, 2014)

Given agents → variable agents: some agents can be replaced sometimes (Torok, 2013)

Homogeneous (undirected interaction) → heterogeneous (directed interaction): ε_i different (Lorenz, 2007)

… …

Theoretical results

Some existing theoretical results: Blondel, Hendrickx, & Tsitsiklis (2009, 2010), Como & Fagnami (2011), Touri & Nedic (2011, 2012), …

✓Convergence: finite-time convergence in HK model and (asymptotical) convergence in DW model

✓Fragmentation: the opinion difference between opinion subgroups (if any) > ε

✓Order preservation in HK model …

✓Consensus if $n \rightarrow \infty$ ……

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4. Our Results
    1. Disagreement: Fragmentation & Fluctuation
    2. Intervention for agreement
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4. Our Results

Agreement or disagreement for simple confidence-based models?

- **A general confidence-based model: opinion fragmentation, separation time (**Physica A 2013, Kybernetika 2014**)**
- **Aggregative long-range interaction: consensus enhancement, opinion fluctuation (**IEEE CDC 2014, Phyisca A 2013, SICON submitted**)**
- **Opinion intervention or noisy model: "consensus" achieved by noise injection (**Automatica submitted; arXiv 2015**)**

Technical challenges

Most OD results based on graph-based models (DeGroot, Friedkin …).

Why confidence-based model?

Importance + fewer results.

Why more technical challenges?

- Strong nonlinearity from bounded confidence + stochastic process → few effective mathematical tools
- Graph is state-dependent → graph theory fails

4.1 Disagreement

Agreement (consensus): all the opinions converge to the same opinion value

Disagreement is very common in OD: two basic phenomena, i.e., fragmentation (convergence; opinion aggregation into clusters/subgroups) and fluctuation (no convergence)

Measurement of disagreement: number of clusters, distance between clusters, and difference between opinion values

$$R_x = \max_{i,j} \left| x_i(t) - x_j(t) \right|$$

Motivation

- **The study of opinion disagreement for general cases;**
- **A general model may cover the traditional HK and DW models (and even some of their variants).**

DW selects a single agent, while HK selects the neighbors → we extend DW model by a selection of multiple agents as candidate to share the opinion in two ways:

- ✓ **local average → short-range interaction → fragmentation**
- ✓ **aggregation → long-range interaction → agreement, fluctuation**

A general model

Short-range multi-selection DW (SMDW) based on local average:

$$x_i(t+1)= x_i(t)+\gamma_i \sum_{j\in S(i)} \alpha_{ij}\, \mathbf{1}_{\{|x^j_{r(t,i)}-x_i|\leq\varepsilon\}}\, (x^j_{r(t,i)}(t)-x_i(t))$$

where $\mathbf{1}$ is the indicator function, ε is the confidence radius; γ_i, $\alpha_{ij} \in (0,1)$; $S(i)$ the selection set with c_i elements.

HK (with c_i as the time-varying number of its neighbors) and DW (with $c_i=1$) can be viewed as a special case of SMDW.

Model analysis

- Written in matrix form: $x(t+1)=W(t)x(t)$, where the elements of $W(t)$ contain the indicator function, which is highly nonlinear.
- $W(t)$ is state-dependent, hard to be analyzed using graph theory.
- Stochastic analysis due to random initial condition and selection process.

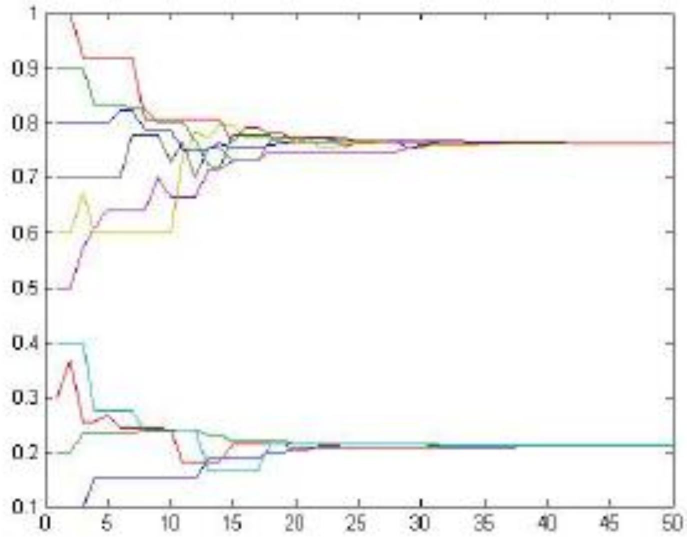
Convergence

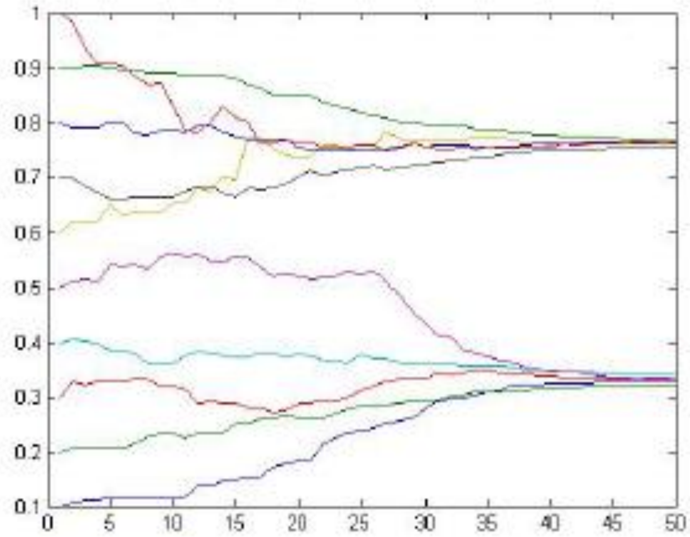
For any $\varepsilon > 0$ **and initial opinions** $x(0)$**, the opinions aggregate to some clusters almost surly (a.s.), that is, either of the following conclusions hold a.s. :**

$$(i)\ \lim_{t\to\infty} |x_i(t) - x_j(t)| = 0,$$
$$(ii)\ \lim_{t\to\infty} |x_i(t) - x_j(t)| > \varepsilon$$

The proof is similar to that for the HK model, but more cases should be discussed

Single selection vs. multiple selection





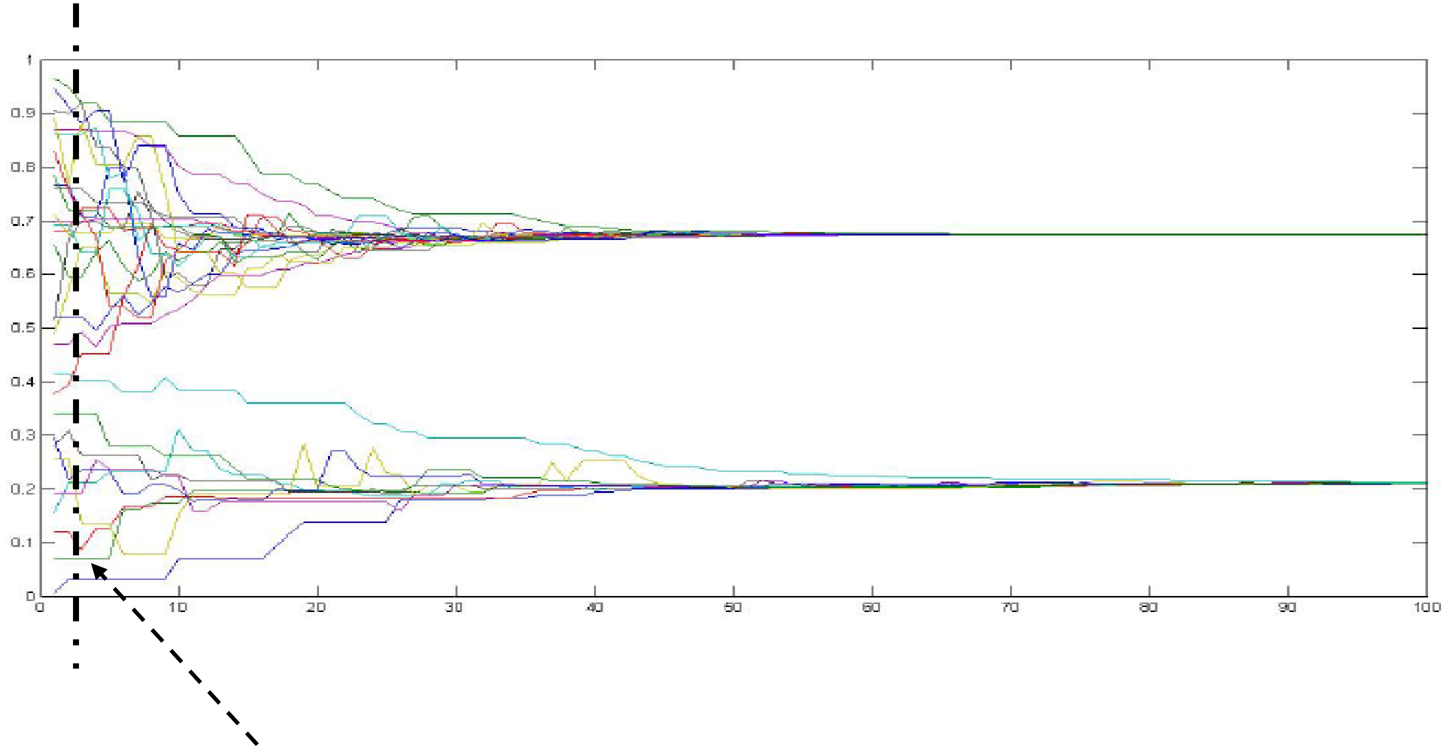
The trajectories in the multiple selection case are smoother with c_i=4

Separation Time

Two steps in fragmentation phenomena: separation + clustering → the opinion values are separated, and then subgroup/cluster aggregation is achieved (i.e., consensus achieved within each cluster)

Separation time T^* is first moment when the steady opinion clusters are formed.

Separation of subgroups



The separation occurs!

The evolution of a DW model: 30 agents with ε=0.4

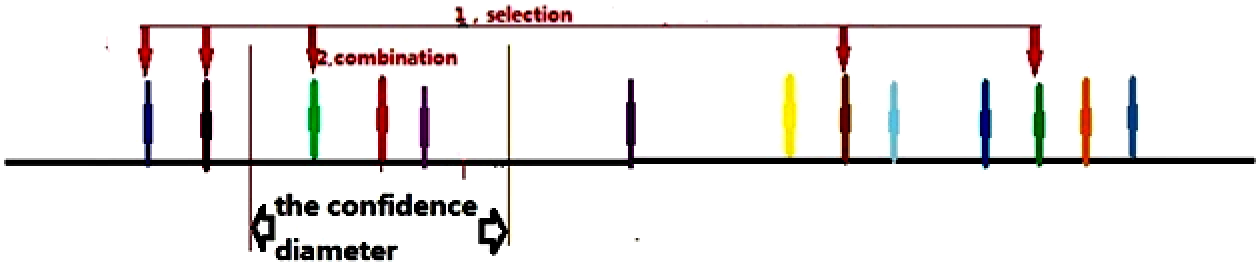
Separation Time Bound

Convergence a.s. but the expectation of separation time *T is bounded by:**

$$E[T^*] \leq 1 + \frac{n^{n-1}}{\varepsilon_0^2(\underline{\gamma}(1-\overline{\gamma}))^{n+1}}$$

which is related to number of agents, confidence bound, and the bound of γ_i

Aggregation interaction



Non-local aggregation: average all the opinions of the selected agents to get an aggregation opinion

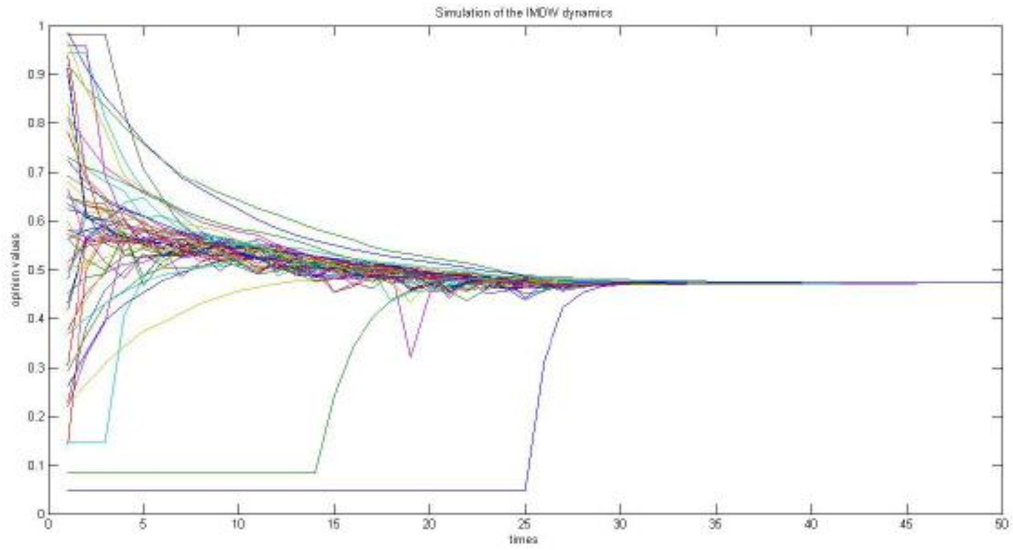
Long-range non-local aggregation model for *n* regular agents:

$$x_i(t+1)=x_i(t)+\gamma_i\,\mathbf{1}_{\{|\sum_{j\in S(i)}\alpha_{ij}(x^j_{r(t,i)}-x_i)|\le\varepsilon\}}\sum_{j\in S(i)}\alpha_{ij}(x^j_{r(t,i)}(t)-x_i(t))$$

where **1** is the indicator function, ε is the confidence radius; γ_i , $\alpha_{ij}\in(0,1)$; $S(i)$ the selection set with c_i elements.

Aggregation → consensus

With c_i>1, the consensus/agreement can be reached a. s. for the non-local aggregation model.



50 agents located in [0,1] with ε=0.4.

Opinion fluctuation

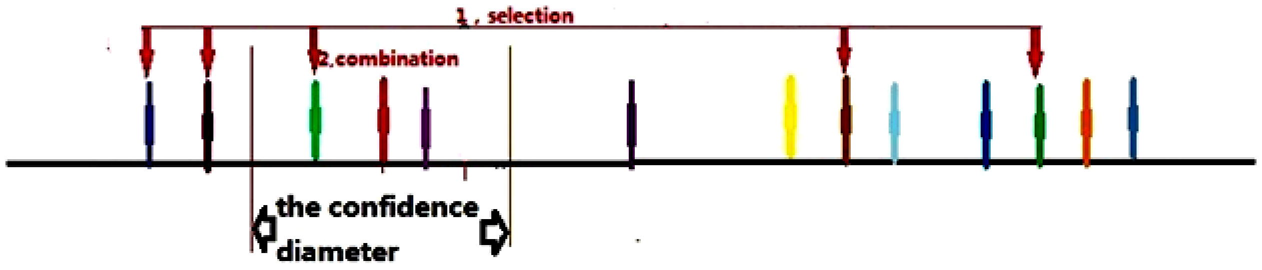
Fluctuation: persistent disagreement between agents, whose opinions never converge to any fixed values → application to voting, fashion, ...

- Kramer (1971): a large swing in voting behavior within short periods

- Cohen (2003): influence on change of political beliefs by parties or organizations

- Acemoglu, et al (2013): graph-based model with stubborn agents (SA), regular ones randomly connected with the SAs

Aggregation + stubborn agents



Still consider the long-range aggregation dynamics:

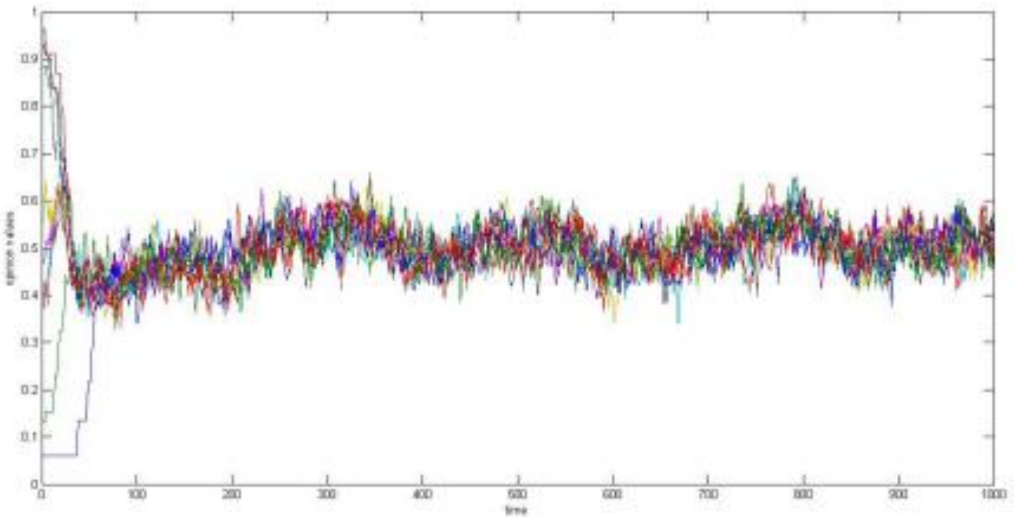
$$x_i(t+1)= x_i(t)+\delta\mathbf{1}\{|\sum_{j\in S(i)} \alpha_{ij} (x^j_{r(t,i)}-x_i)|\leq \varepsilon_0\}\sum_{j\in S(i)} \alpha_{ij} (x^j_{r(t,i)} (t)-x_i(t))$$

where **1** is the indicator function, ε_0 the confidence radius; δ, $\alpha_{ij} \in (0,1)$; $S(i)$ the selection set with c agents.

In the network, n regular agents and m stubborn agents with fixed values as 1 or 0.

Critical bound

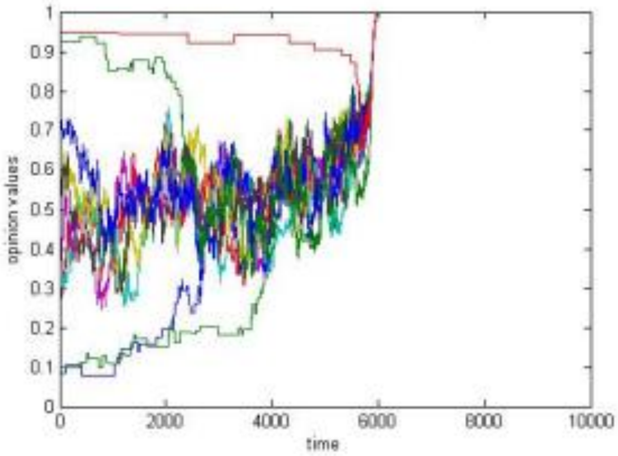
Fluctuation phenomena with taking c=6, ε_0=0.2



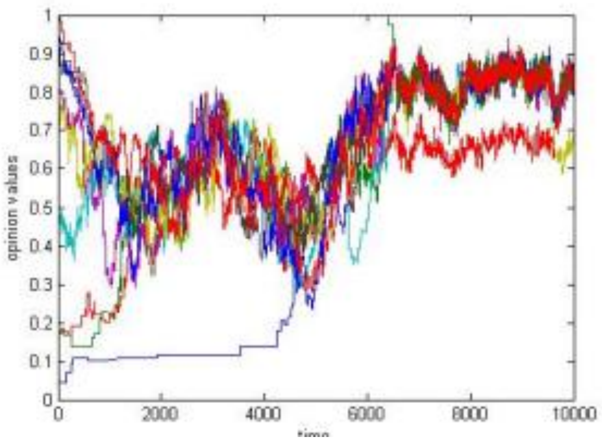
Result: fluctuation almost surely if and only if $\varepsilon_0 \geq \frac{1}{c}$

Small bound

If $\varepsilon_0 < 1/c$, convergence may happen, and the probability for the opinions converge to either 0 or 1 (opinion value) is larger than $2\varepsilon_0^n$.



convergence



fluctuation

Fluctuation strength

Take $\delta \in (0,0.5)$ and $\varepsilon_0 \geq \dfrac{1}{c}$

Fluctuation strength can be measured by

$$R_{\mathbf{x}}(t) = \max_{i,j\in\mathcal{M}} |x_i(t) - x_j(t)|$$

Its estimations are given as follows:

$$\overline{\lim}_{t\to\infty} R_x(t) \leq \delta\ Q \qquad \underline{\lim}_{t\to\infty} R_x(t) \geq \frac{\delta}{c}$$

where Q is a function of system parameters (quite complicated).

4.2 Opinion Intervention

- **Intervention is important for social studies, to make the society stable, or unstable, or make it transfer to some specific states … …**
- **Intervention never stops in reality.**
- **Intervention design related to: control and optimization, swarm intelligence (ants → people), learning and evolution (with supervisor) …**

Intervention → control

- Related to control, but modern control theory cannot be applied! Cannot control the society as mechanical systems with enough actuators or power
- New control methods in soft, covert, simple, and indirect ways → a complicated procedure involved with networks
- **A basic problem**: reduce or eliminate social disagreement by intervention (because disagreement may yield social instability …)

Noise Injection

Motivation: inject noise to increase the consensus probability; consensus analysis for noisy confidence-based model

Consider a modified term by injecting noise to selected agents:

$$x_i^*(t) = \begin{cases} |\mathcal{N}(i, x(t))|^{-1} \sum_{j \in \mathcal{N}(i,x(t))} x_j(t) \quad + \xi_i(t+1), \\ \qquad \text{if } i \in \mathcal{I}, \\ |\mathcal{N}(i, x(t))|^{-1} \sum_{j \in \mathcal{N}(i,x(t))} x_j(t), \quad \text{if } i \in \mathcal{V} \backslash \mathcal{I}, \end{cases}$$

where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the set of agents, and $\mathcal{I} \subset \mathcal{V}$ is the set of the noise-injected agents. The neighbor set is defined by the confidence bound ε

Noisy HK model

Consider the HK model with additive noise:

$$x_i(t+1) = \begin{cases} 1, & x_i^*(t) > 1 \ , \\ x_i^*(t), & x_i^*(t) \in [0,1] \\ 0, & x_i^*(t) < 0 \ . \end{cases}$$

where the noises $\{\xi_i(t)\}_{i\in\mathcal{V},t\geq 1}$ are mutually independent, with

$$\begin{aligned} &|\xi_i(t)| \leq \delta \ \text{(where } \delta \text{ is a positive constant);} \\ &E\xi_i(t) = 0; \\ &E\xi_i^2(t) = \sigma_i^2(t) \geq c\delta^2 \text{ for a constant } c \in (0,1] \end{aligned}$$

Quasi-consensus with noise

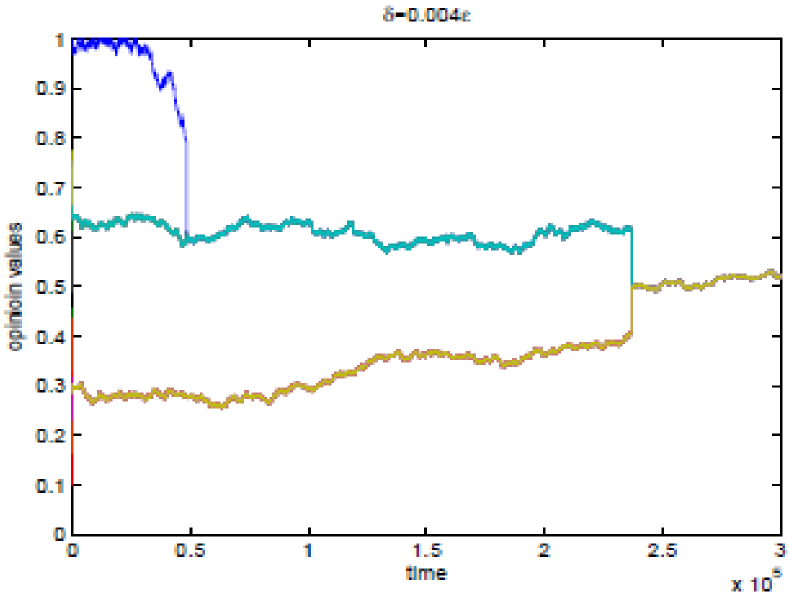
Noise injection to OD may be simply realized by starting rumors or spreading slanders, etc

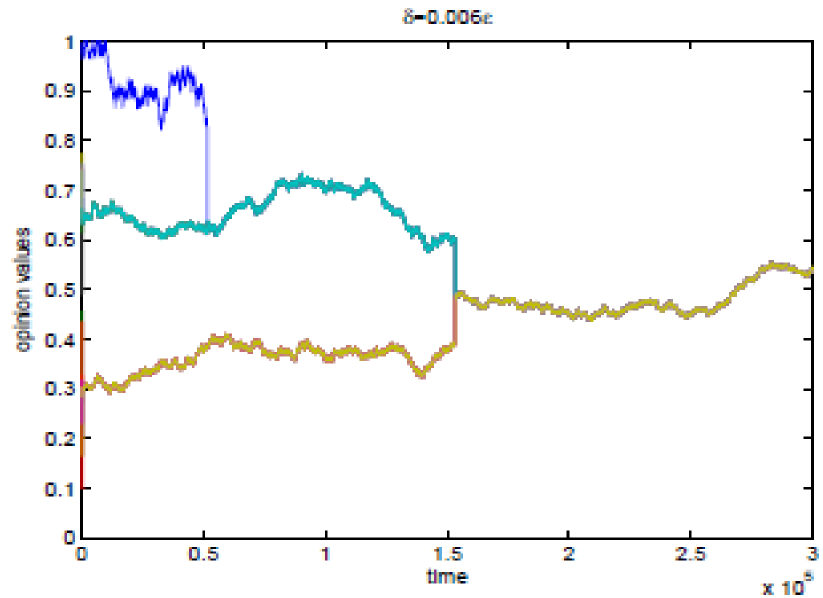
Result 1： If $P(|\xi_i|\leq\delta=\varepsilon/2)=1$， then the opinions almost surly achieve quasi-consensus ("consensus" with error less than ε) in finite time.

Result 2: Take $\varepsilon\in(0,1/3)$. If $P(\xi_i>\varepsilon/2)>0$ and $P(\xi_i<-\varepsilon/2)>0$, then the system cannot achieved quasi-consensus.

These results are strictly proved based on careful stochastic analysis (due to random initial condition)

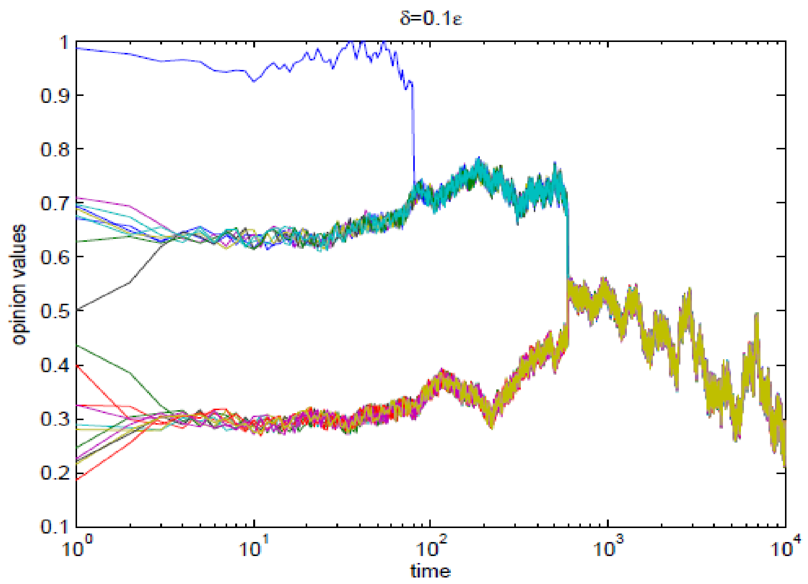
Simulation

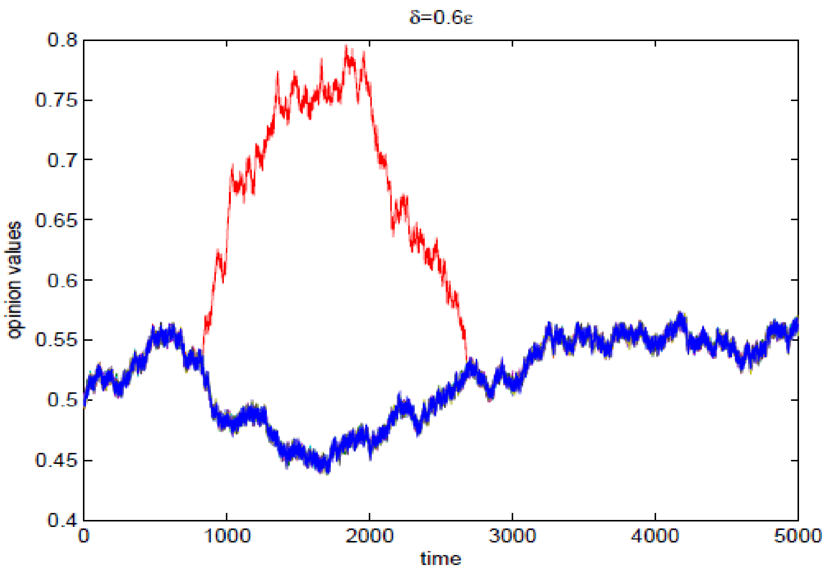




Similar phenomena are also found in a noisy HK model by Pineda et al (2013), without strict mathematical analysis.

Simulation (2)





Large noise may spoil the quasi-consensus as shown in the second figure

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5. Conclusions

- Good time to give mathematical models for the analysis, prediction, and intervention of social behaviors
- Simple confidence-based models → opinion disagreement (fragmentation, fluctuation), or a simple intervention for opinion "consensus" by injecting noise.
- Next: blend of confidence-based and graph-based models, models with evolved confidence/trust , …

New Era → New …

- Many social problems → new models and methods → new control theory and technology ≈ model-based analysis/design + data-based technology
- Underlying mechanics of social network → social learning and swarm intelligence methods

- Engineering + social studies → new social results based on engineering ideas, new engineering methods inspired by social ideas

Thank you！

☺