

ENCE 201 Midterm 2, Open Notes and Open Book

Name : _____

E-mail (print neatly!): _____

Exam Format and Grading. This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are two questions. Partial credit will be given for partially correct answers, so please show all your working.

Note: Please see the **class web page for instructions on how to submit your exam paper.**

Question	Points	Score
1	20	
2	20	
Total	40	

Question 1: 20 points.

This question covers solution of matrix equations using Gauss Elimination. Consider the matrix equations $Ax = b$, where:

$$\begin{bmatrix} 0 & 7 & 3 \\ 3 & 0 & 7 \\ 7 & 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \\ 17 \end{bmatrix}. \quad (1)$$

[1a] (4 pts). Compute the $\det(A)$?

[1b] (4 pts). Based on your solution to part 1a, what can you say about: (1) the rank of the system of equations, and (2) the number of solutions to the matrix equations?

[1c] (4 pts). In terms of “complexity of matrix structure” and “row operations,” what are the goals of the method of Gauss Elimination? Why does the method work?

[1d] (8 pts). Use the method of Gauss Elimination with pivoting to compute the solution to equation 1. This is a hand calculation, so show all of your working.

Question 2: 20 points

[2a] (5 pts). Derive the Newton-Raphson formula:

$$x_{n+1} = x_n - \left[\frac{f(x_n)}{f'(x_n)} \right] \quad (2)$$

Hint : Start off by writing down a Taylor's series expansion for $f(x+h)$. State all of your assumptions.

[2b] (5 pts). Consider the function

$$y(x) = \sin^2(x). \quad (3)$$

Show that the Newton-Raphson update formula can be written as:

$$x_{n+1} = x_n - \frac{1}{2} \tan(x_n). \quad (4)$$

Be sure to show all of your working.

[2c] (5 pts). If $\dot{g}(x)$ and $\ddot{g}(x)$ are the first and second derivatives of $g(x)$, respectively, and

$$f(x) = \left[\frac{g(x)}{\dot{g}(x)} \right] \quad (5)$$

show that the modified Newton-Raphson formula is given by:

$$x_{n+1} = x_n - \left[\frac{g(x_n)\dot{g}(x_n)}{\dot{g}(x_n)\dot{g}(x_n) - g(x_n)\ddot{g}(x_n)} \right] \quad (6)$$

[2d] (5 pts). Use a starting value $x_o = \pi/4$ and the modified Newton Raphson Formula to find an improved estimate of the root of the polynomial:

$$y(x) = \sin^2(x). \quad (7)$$

Note: Do no more than 1 iteration !!.