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Complex variable methods in the modelling and control of distributed parameter systems

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We describe certain advances in modeling and control of linear, time invariant, distributed parameter systems, based on some sophisticated results from complex variables and transforms. For the purposes of this paper distributed parameter systems means infinite dimensional systems in general; i.e., systems governed by partial differential equations, systems governed by integrodifferential equations, systems governed by hereditary ordinary and partial differential equations, any combination of the above, and systems governed by some more complicated dynamical models. The reason for this generality is twofold: (a) we will describe examples from large flexible structures and controlled wave propagation where such generality is necessary; (b) we want to demonstrate clearly the power of the proposed methods in dealing with models considerably beyond those described by partial differential equations (p.d.e.).

We describe the foundations of the frequency domain methods beginning from p.d.e. models and developing their complex variables analogs using Laplace and Fourier transforms, including multidimensional ones. We shall primarily treat the important for applications case of finite-dimensional input and output signal spaces.

We then describe some simple design and control problems that can be easily solved using complex variables methods. In particular, we shall describe dynamic stabilization of beams with delay in the feedback loop of a stabilizing, boundary damping feedback, and the solution of quadratic regulator problems involving multidimensional wave equations. We will then describe the necessary arguments for obtaining these results by using p.d.e. methods for purposes of comparison. A complete formulation of a general regulator problem in the frequency or transform domain is provided. Based on this we describe the solution of the dynamic compensation problem, and a generalization of the Youla-Bojiorno-Wiener-Hopf design method.

In these developments we discuss the key role played by corona type theorems in the control and design problems described here. Starting from the famous and classic Carleson corona theorem, we give modern proofs of corona theorems in several complex variables and over domains of interest for the control of distributed parameter systems. We show how conformal mapping can be used to advantage and how all this in intricately related to the so-called delta bar problem over various domains. In particular, we discuss and develop the necessary factorizations for control problems by using these corona theorems systematically to obtain Bezout identities in the various algebras involved.

Finally, we discuss numerical and computational methods for CAD of such control systems based on the mathematical techniques described.