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# Nonlinear Filtering Asymptotics, Large Deviations and Observers for Nonlinear Systems

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## Abstract

This paper presents a design for an observer for the nonlinear control system

$$\begin{aligned} \dot{x} &= f(x,u), \qquad x(0)=x_0, \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n, u \in \mathbf{R}^m, |u_i| \leq 1 \ i=1,\ldots,m$ and $y \in \mathbf{R}^p$. The initial condition $x_0$ is unknown.

The *observer problem* consists of recursively computing an estimate $z(t)$ of $x(t)$ for which the error decays to zero as $t \to \infty$ , that is, to design a system

$$\begin{aligned} \dot{m} &= F(m,u,y), \qquad m(0)=m_0, \\ z &= G(m) \end{aligned} \tag{2}$$

such that

$$\lim_{t\to\infty} |x(t)-z(t)| = 0 \tag{3}$$

for all $x_0$ in a suitable class $I$. Here $I$ represents a priori knowledge concerning the initial condition $x_0$.

Baras and Krishnaprasad [4] have proposed a method for constructing an observer as a limit of nonlinear filters for a family of associated filtering problems, parameterised by $\epsilon > 0$. More recent work in this direction is presented in [1-3]. We study the asymptotic behaviour of the corresponding unnormalised conditional densities $q^\epsilon(x,t)$ as $\epsilon \to 0$, and obtain the asymptotic formula

$$q^\epsilon(x,t) = exp\left(-\frac{1}{\epsilon}(W(x,t)+o(1))\right), \tag{4}$$

as $\epsilon \to 0$, where $W(x,t)$ is the value function corresponding to a deterministic optimal control problem, namely that arising in deterministic estimation.

Hijab has studied this asymptotic estimation problem, and obtained a WKB expansion when $W(x,t)$ is smooth. This identifies the limiting filter as Mortensen's deterministic or minimum energy estimator. In addition, Hijab has proved a large deviation principle for the conditional measures for the filtering problem. We extend Hijab's large deviation result by allowing random initial conditions, and observe that the resulting variational problem (c.f. action functional) is exactly the optimal control problem mentioned above.

The asymptotic formula for the unnormalised conditional densities and the large deviation principle for the unnormalised conditional measures characterise the limiting filter in terms of the deterministic estimator.

We prove the following result for our observer design: provided that we have some knowledge of $x_0$ (in the form $|x_0 - z_0| < p$, where $z_0$ is the initial estimate) and assuming that (1) satisfies a *detectability* condition, then the observer estimate $z(t)$ converges exponentially to the system trajectory $x(t)$ as $t \to \infty$. The radius of convergence $p$ depends on the nonlinearities in the dynamics and observations as well as on certain design parameters. For a certain class of systems, no knowledge of $x_0$ is required.

The results described here are a summary of our recent results in [1-3], where we refer the reader for further details. We remark that these designs do not involve coordinate transformations, canonical forms, local linearization, etc., and seem robust when compared with other designs. However, the designs do involve solving Riccati equations and computing certain matrices and constants.

## References

[1] J.S. Baras, A. Bensoussan, and M.R. James, *Dynamic Observers as Asymptotic Limits of Recursive Filters: Special Cases*, SIAM J. Applied Math., Vol. 48, No. 5, October 1988, pp. 1147-1158.

[2] M.R. James and J.S. Baras, *Nonlinear Filtering and Large Deviations: A PDE-Control Theoretic Approach*, Stochastics, Vol. 23, pp. 391-412, 1988.

[3] M.R. James and J.S. Baras, *An Observer Design for Nonlinear Control Systems*, Proceedings of the 1988 INRIA Conference on Optimization of Systems, Antibes, France, 1988.

[4] J.S. Baras and P.S. Krishnaprasad, *Dynamic Observers as Asymptotic Limits of Recursive Filters*, Proc. of IEEE 21st CDC, 1982, pp. 1126-1127.